Do Developmental Mathematics Courses Develop the Mathematics? Addressing Missing Outcome Problem in Regression Discontinuity Design*

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Abstract

Many students are unprepared for college-level math in spite of many attempts to improve the math skills of high-school students. In community colleges, developmental mathematics courses are designed to help those students make up for the gaps in high-school math. However, there are few studies on the effect of developmental mathematics on mathematics achievement despite the vast quantity of research on the courses' effects on various outcomes. Developmental mathematics consists of various courses in a tight sequence where course assignments are determined by a rigid placement rule based on students' test scores, and in which students must master the assigned course before taking the next level of math. A course's effectiveness can be measured by the letter grade or other test scores in its subsequent course. However, such an effect is difficult to investigate because of missing outcome problems; achievement in the subsequent course is only observed for those who enrolled and finished it. Enrollment may be affected by assignment to a prerequisite course since those assigned to the prerequisite are less likely to enroll in the subsequent course compared to those assigned directly to the subsequent course. In regression discontinuity design (RDD), usual methods such as the control function approach cannot address these missing outcome problems as the outcome's propensity to be observable is also discontinuous. Applying a bounding approach in RDD, this study partially identifies the causal effects of developmental mathematics, and computes their bounds. Using the data from a community college in California, I find that assignment to developmental courses would increase achievement and learning efficiency in the subsequent math courses.

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1 Introduction

The importance of mathematics cannot be emphasized too much. The importance of mathematics taught in secondary school has been shown in many studies of wages and productivity. Among all the subjects taught in secondary school, mathematics and science matter most to the productivity of individuals and national economies. Individual achievement in high-school mathematics[1] correlates with the wages of high-school graduates. The estimated effects of mathematics achievement on individual's productivity are shown to be stronger than those of any other subjects such as English reading or writing (Rose and Betts, 2004; Goodman, 2012; Altonji, Blom, and Meghir, 2012)[2]. Also, the average student performance in mathematics[3] has been shown to contribute more to economic growth than performance in any other subject when one controls for quantity of education or years of schooling and restricts results to the developed countries (Hanushek and Kimko, 2000; Barro, 2001; Hanushek and Woessmann, 2008).

Differences in math achievement might contribute to the wage gap, and lower performance in math might impair economic growth. In fact, in the U.S, the lower the economic and social status of an individual student, the lower his or her math achievement is likely to be. In addition, American students perform more poorly than their peers in other industrialized countries on standardized math exams. Aware of the importance of math in high school and of the weakness of education in the U.S, many studies have focused on the determinants of high-school math achievement and how to elevate them through reforms in the graduation requirements and curriculum standards. In particular, recent studies have shown that algebra courses play an important role in math achievement at the secondary level (Schneider, Swanson, and Riegle-Crumb, 1997; Gamoran and Hannigan, 2000), and hence are a key factor in performance in postsecondary-level math (Adelman, 2006; Long, Iatarola, and Conger, 2009; Long, Conger, and Iatarola, 2012), and finally, in the outcomes of labor markets (Rose and Betts, 2004; Goodman, 2012; Altonji, Blom, and Meghir, 2012). Moreover, there have been made many attempts to improve low achievement in algebra courses; for example, an acceleration of algebra 1 and a universal algebra policy were implemented in California. Yet in spite of many interventions at the early stage in secondary school, many students graduate from high school with insufficient math skills.

[1]As measured by i) indicator variables of whether to complete the advanced math course and ii) the number of math courses completed by the individual.

[2]At first, Altonji (1995) makes an attempt to systematically show the effects of high-school curriculum on wages. But their effects are shown to be weak in this result, though mathematics has more effects than any other subject. Similarly, Levine and Zimmerman (1995) try to estimate such effects, restricting their study to math and science courses, and their estimation results are stronger than Altonji (1995)'s.

[3]As gauged by scores on standardized and international tests such as PISA international test.

To those students, community colleges have generously granted a second chance through the open admission policy and a sequence of developmental mathematics courses. Many students are assessed as lacking skills in algebra or high-school math, and they are banned from college-level mathematics such as trigonometry and calculus. Instead, they are assigned to any one of a variety of courses in developmental mathematics.

Only about 9% of community-college students were assigned to non-developmental mathematics courses in California (Serban et al., 2005), and over 40% of such students need to be educated at the high-school level nationwide (Adelman, 2004). Since most of these students come from minority backgrounds, the use of developmental mathematics may be an excellent way to decrease the wage gap by improving their math skills. However, there is one respect in which developmental mathematics offered at community colleges is identical to high-school math: material. Some opponents argue that this is a typical example of waste of the public resources and that providing almost costless second chances can demoralize high-school mathematics education.

Despite its prevalence and the controversy surrounding it, few elaborate studies have paid attention to the effect of the developmental mathematics offered at community colleges. Most studies of community colleges' developmental mathematics are descriptive analyses. There are two remarkable studies of general developmental education where the effect of developmental mathematics on various outcomes is estimated (Calcagno and Long, 2008; Martorell and McFarlin, 2011). To address the endogeneity or self-selection problem, both exploit the assignment rule based on test scores for regression discontinuity design. One drawback exists– these studies pay attention not to academic performance in mathematics itself, but to general long-term outcomes such as credential attainment, transfer to 4-year institutions, and graduation with a degree. They are far from the direct measure of math achievement. Both studies neglect the estimation of the fundamental function of developmental education, which is that a developmental course should help a student make up for the lack of that course and be ready for the next-level course.

Instead of long-term outcomes, I examine a short-term outcome: the performance in a math course[4]. I investigate the effect of developmental math on the student's performance in the next-level math course. For each developmental math course, its corresponding outcome is defined by i) the grade point average (GPA) on its subsequent course and ii) the time to complete that subsequent course. The corresponding control group is the students assigned

[4]For a developmental English program, Moss and Yeaton (2006) use the letter grade on the first college-level course as the measure of performance. The treatment is the course of one level below the college English course. They use regression discontinuity design to address the endogeneity of course assignment. However, their way of interpreting the estimated effects is uncertain and they ignore the missing outcome problems.

directly to this subsequent course. Because of my interest in improving algebra achievement, I choose to look at the performance in two algebra courses: elementary algebra and intermediate algebra. Their corresponding treatments are their prerequisite courses, pre-algebra and elementary algebra respectively. Thus, studying the effect of the developmental math sequence program is equivalent to studying the effect on the one specific course of its prerequisite course.

Using a longitudinal dataset of one community college in southern California, this study tries to estimate the effect of a developmental math course the performance in its subsequent math course. I also rely on regression discontinuity design, since the placement policy of the chosen community college assigns students to the specific course based on their scores on the assessment test. Contrary to the other studies using regression discontinuity design, however, one serious difficulty arises here: the missing outcome problem. Achievement in a math course can be observed and defined only if a student finishes/completes it. A conventional approach to the missing outcomes (or the sample selection problem) is to generate control function variables to correct the bias from the missing outcomes (or sample selection) by exploiting the exclusion restriction or instrumental variables. But such an approach cannot be used here, since the observability of the outcome is discontinuous at the cutoff point[5]. Due to the structure of developmental sequence, the observability differs drastically between the control and treatment groups, even when the study is restricted to the students whose test scores are close to the cutoff point. Those who are assigned to the prerequisite course are significantly less likely to proceed to the next-level course and finish it, because the assignment to the prerequisite course requires longer time and higher opportunity costs.

The main contribution of this study is to compute the bounds for the treatment effect of the prerequisite math course on the next math course in a sequence of developmental math courses, addressing the missing outcome problem in regression discontinuity design, a problem that cannot be handled by the conventional method. By modifying the bounding approach in the case of missing outcome problems (Horowitz and Manski, 1995, 2000; Lee, 2009) into regression discontinuity design, I can solve the problem of the structural difference in observability between the control and the treatment groups.

Applying this bounding approach in regression discontinuity design, I find that assignment to developmental courses would increase achievement in the subsequent math courses and reduce the time to complete the main course. The estimated effects of some developmental courses are found to be insignificant, but these results are confounded with the discontinuity in high-school math achievement, which is measured by multiple measure points

[5] Another reason is that reliable exclusion restrictions or instrumental variable cannot be found.

calculated by the selected community college. Adjusting for this discontinuity, the insignificance of estimates is shown to be due to downward bias. This result contrasts with the ineffectiveness of developmental mathematics on long-term outcomes such as transferring to four-year colleges and labor-market outcomes (Calcagno and Long, 2008; Martorell and McFarlin, 2011).

The rest of the paper is organized as follows. Section 2 describes developmental education and the sequences of mathematics courses, and gives a summary of the previous literature. Section 3 begins with a brief description of the estimation method in regression discontinuity design. It is followed by the explanation of the nature of the missing problems which appear in this study. Regarding the missing outcome problem and regression discontinuity design, the bounding approach is derived in the context of this study. In Section 4, I describe the sample and the outcomes used for the analysis. Section 5 reports the results from the empirical analysis of the chosen community college, and Section 6 discusses the validity tests for regression discontinuity design. Section 7 concludes.

2 Developmental Math Program in Community Colleges

2.1 Developmental Education in Community Colleges

One of the primary roles of community colleges is to offer developmental, remedial or preparatory education (Cohen and Brawer, 2008; Grubb, 2004). The definition of developmental course work is straightforward. Developmental education in community colleges is defined as coursework below college level offered at postsecondary institutions[6]. In the process of developmental education, students learn the academic skills and knowledge that should have been acquired in high school.

The reason developmental education is so widely practiced in community colleges is that such colleges adopt an open admission policy. The open admission policy lets in anyone who wants to enroll in a community college without entrance requirements. Due to the open admission policy, however, there exists wide variation in students' academic preparation. In particular, the most poorly prepared group of high school graduates is in community colleges and they want to go to four-year colleges. Developmental education is designed to give those students the chance to make up their deficiencies of skill. Due to developmental education, community colleges have been called the most important "second-chance" institutions and

[6]Developmental or remedial programs in K-12 are different from the ones used by community colleges or postsecondary schools. For example, summer school and grade retention are designed to help disadvantaged students to reach the minimum standard at those schools.

"people's colleges" (Grubb, 2004). Most of community colleges offer developmental education in two fundamental subjects: English reading/writing[7] and mathematics.

In order to determine whether a student should enroll in developmental coursework, he or she should be assessed through placement tests when entering the community college. Placement tests assess how much students learned in high school and determine what courses are appropriate for them. The placement test can reveal how many students need developmental education. According to Serban et al. (2005), only about 9% of students were assigned to non-developmental mathematics courses and about 27% of students were assigned to any non-developmental English courses in the California Community College System[8].

2.2 A Sequence of Mathematics Courses

In community colleges, developmental mathematics takes priority over other developmental education. On average, community colleges typically offer one more developmental course in mathematics than in English reading or writing (Parsad, Lewis, and Greene, 2003). At the level of individuals, Adelman (2004) finds that the proportion taking only developmental mathematics is at least 25% higher than the proportion taking other developmental courses among freshmen enrolled in any development education at community colleges. Moreover, a developmental mathematics course costs more than other developmental courses or regular college courses because of the large number of students in the developmental courses and the very high rates of withdrawal.

Developmental mathematics courses are differentiated and sequentially organized. Typically, mathematics is organized as a cumulative and linear sequence of topics. These sequences are designed so that a student must master certain concepts and skills in an assigned course before advancing to a course one level higher. If the student does not master the given concepts and skills, he or she cannot enroll in higher-level courses such as college-level mathematics. Thus, individual courses are part of a larger unified subject that is minimally necessary for learning college-level mathematics. These courses are taught with progressive levels of difficulty throughout the developmental sequence. The sequence of developmental mathematics courses is organized hierarchically by topic and ability tracking.

Most of these properties of developmental mathematics are shared with secondary schools' mathematics sequences[9]. The courses taught in the developmental mathematics sequence of community colleges are equivalent to the ones in the high-school mathematics sequence.

[7]English as a second language (ESL) programs can belong to developmental reading/writing education.

[8]Nationwide, more than 40% of community-college students need to be educated at the high-school level (Adelman, 2004).

[9]High-school mathematics sequences are described in Schneider, Swanson, and Riegle-Crumb (1997).

The most common courses of the development sequence are 1) arithmetic, 2) pre-algebra, 3) elementary algebra, and 4) intermediate algebra. Arithmetic is generally the lowest level of mathematics. It reviews the fundamentals of arithmetic that are essential to success in the other mathematics courses, and it covers the material of pre-8th grade mathematics. A pre-algebra course bridges the gap between arithmetic and general algebra. An elementary algebra course is for those who have had no algebra 1 in high school or whose preparation is deficient, while an intermediate algebra covers the material of algebra 2 in high-school math. The distinctive feature of developmental math courses offered in community colleges is that they teach students high-school mathematics within the one unique sequence. However, taking high-school math in community colleges could be a waste of time and resources for some students who are assigned to it in spite of having already taken it.

2.3 Algebra

Especially in high-school curriculum, algebra courses are regarded as the most important. Intermediate algebra or algebra 2 is a key factor in academic achievement at the college level nationwide (Adelman, 2006; Long, Iatarola, and Conger, 2009; Long, Conger, and Iatarola, 2012) and in the outcomes of labor markets (Rose and Betts, 2004; Goodman, 2012; Altonji, Blom, and Meghir, 2012). The largest gains occur at algebra 2. Although taking elementary algebra or algebra 1 (or pre-algebra) alone does not guarantee any improvement in readiness for college-level math or in labor-market outcomes, it is identified as the gateway to success in the algebra sequence. Many concerns are raised about algebra courses as a serious equity and civil-rights issue, and hence many policies accelerating algebra instruction into middle school[10] have been implemented to enhance student success in algebra (Gamoran and Hannigan, 2000; Loveless, 2008; Clotfelter, Ladd, and Vigdor, 2012).

Unlike accelerating algebra instruction, algebra courses in developmental mathematics sequences offered in community colleges may be a kind of late intervention. They are intended for students who are deficient in algebra or who have not attempted it in secondary school. The developmental sequence intends to help those students not only prepare for college-level math but also develop skills and knowledge of algebra. For example, knowledge of elementary algebra is weak and the rate of completion of intermediate algebra is low among the students enrolled in community colleges in California (Serban et al., 2005), though early algebra-taking rates exceeded 59% in California and it is higher than in the other states[11] (Loveless, 2008).

[10] A kind of early intervention.

[11] This result may come from the fact that the mathematics requirement for graduation is not strict in California. Completing one year of algebra 1 is the minimum requirement, though the other course is necessary for postsecondary success.

Many attempts to improve algebra skills in high school have been made, but they have been shown to be ineffective (Clotfelter, Ladd, and Vigdor, 2012). With little effect from this early intervention, late interventions such as developmental education in community colleges can play an important role. The disadvantage of late interventions is that they would doubly waste resources unless they are effective in improving mathematics skills in those who did not benefit from intervention at an early age. So far, studies have not investigated whether the developmental mathematics sequence has assisted students who are deficient in algebra to make up for their lack of knowledge.

2.4 Previous Literature and their Limitations

While most early studies of developmental mathematics suffer from endogeneity or selection problems because math enrollments are not randomized, the recent studies are good at addressing selection bias by use of regression discontinuity design (Calcagno and Long, 2008; Martorell and McFarlin, 2011) or instrumental variable estimation (Bettinger and Long, 2009). One drawback of these recent studies is that all they are interested in only general academic outcomes such as credential attainment, transfer to four-year institutions, or graduation with a degree, which are a little far from any direct measure of mathematics achievement[12]. They do not pay attention to academic performance in mathematics itself. Moreover, they do not consider the detailed structure of developmental sequences which assign students to various levels. In contrast to the previous studies, Bailey, Jeong, and Cho (2010) examine the relationship between the initial assigned mathematics and an interesting outcome– the highest level that a student reaches in the structure of the developmental sequence. They show that the lower the level on which a student is placed, the less likely he or she is to complete the developmental sequence, but this cannot be firm evidence of causality due to the limitation of their descriptive method.

The essence of any developmental mathematics sequence is that a course in the sequence should be designed to help a student make up for his or her own lack and be ready for the next-level course. Any given course is the prerequisite course to the next-level course in any developmental sequence. Most of the studies of developmental mathematics have not investigated whether the aim of developmental math programs is attained or not, i.e., whether the assigned courses in a developmental math sequence are effective in developing skill in their subsequent courses or not. It seems that estimating the effect of the developmental math sequence program is equivalent to estimating the prerequisite course's effect on its

[12] The studies of developmental mathematics using the other measures of outcomes and other methodologies are well summarized in Bahr (2008), but most are descriptive analyses.

subsequent course[13].

Like other studies using regression discontinuity design, the enforced assignment rule using test scores generates a good regression discontinuity design to address concerns regarding selection into courses on the basis of unobserved characteristics, when studying the effect of the prerequisite course on the achievement in its subsequent course. Although regression discontinuity design controls the endogeneity problem in the study of math achievement itself, one serious difficulty arises: the missing outcome problem. Many of those who were assigned to a low level math course do not proceed to the next level, even when they completed their assigned course[14].

The lower the level to which a student is assigned, the more time he or she spends there and the more it costs him or her to be in a community college. Those students are more likely to leave. In addition, easy access to community colleges through the open admission policy and low tuition makes it easier not only to enter and but also to leave the institutions. Restricting the sample to those students who finish/complete the main course would introduce a sample selection problem.

A missing outcome (or sample selection) problem that occurs in the study of community colleges' developmental mathematics sequence is much more difficult to handle than one that arises in the other studies because the assignment itself creates a discontinuity in missing outcome proportions between the treatment group and the control group. In this case, it is impossible to correct or adjust the bias problem from the sample selection in the context of regression discontinuity design, even if any exclusion restrictions can be found. In the next section, I suggest how to address missing outcome (or sample selection) problems in regression discontinuity design.

3 Empirical Strategy: Bounding Approach in Regression Discontinuity Design

3.1 Regression Discontinuity Design

This subsection presents an econometric model in the regression discontinuity design. It is understood in the context of Rubin's potential outcome model. I simplify the situation of community colleges, assuming that there is one main math course and one prerequisite course; e.g., the main math course is elementary algebra and the prerequisite is pre-algebra.

[13] In this study, algebra courses and their relevant courses are of interest.

[14] The same pattern is frequently observed in the high-school math sequence. (Schneider, Swanson, and Riegle-Crumb, 1997).

The latter is a treatment to improve achievement in the main course. The treatment group consists of the students who are assigned to the prerequisite, while the control group consists of the students who are assigned to the main course directly. $Y_{i,1}$ is what a given student i would achieve in the main course if he or she were assigned to the prerequisite. $Y_{i,0}$ is what a given student i would achieve in the main course if he or she were assigned to the main course. Both outcomes cannot be observed simultaneously for the same student i. Denote a binary indicator for taking the prerequisite mathematics by T_i.

$$T_i = \begin{cases} 1 & \text{if a student } i \text{ is assigned to the prerequisite} \\ 0 & \text{Otherwise} \end{cases}$$

Then the observable achievement in the main math course Y_i for a student i is expressed in the following equation.

$$Y_i = T_i Y_{i,1} + (1 - T_i) Y_{i,0} \tag{1}$$

The individual causal effect of the prerequisite is the difference in two potential outcomes, $\tau_i = Y_{i,1} - Y_{i,0}$. Then the average treatment effect is identified as the difference in two conditional expectations, $E(\tau_i) = E(Y_{i,1}) - E(Y_{i,0}) = E(Y_{i,1}|T_i = 1) - E(Y_{i,0}|T_i = 0)$ if T_i is randomly assigned, i.e., $(Y_{i,1}, Y_{i,0}) \perp T_i$. However, the prerequisite course is not randomly assigned in the real world. This causes the problem in identification of the causal effect of the prerequisite course.

Regression discontinuity design takes advantage of the cutoff policy rules to estimate the causal effect of the prerequisite course on the achievement in the main math course. A usual assignment rule is the cutoff policy based on the student's assessment test score. A student is assigned to a prerequisite course if her or his score on the assessment test is less than the exogenously determined cutoff score. When looking at the students whose test scores are close to a preset cutoff point, regression discontinuity design is similar to a random experiment in which a prerequisite course is assigned by a randomization process.

Let X_i be student's assessment test score. The cutoff point c is set by the community college. Then the treatment or the assignment of the prerequisite course, T_i is a deterministic function of student's test score X_i in the following way[15]: $T_i = 1(X_i < c)$.

The assignment, however, is not random as the test score may be correlated with the educational outcome. Since students who must take a prerequisite may differ from those who

[15] It is implicitly assumed that all students would always follow the placement result. A student who is assigned to the prerequisite course must take the prerequisite and always takes it.This case is called a sharp regression discontinuity.

are directly assigned to the main math course, the comparison of achievements in the main math course between the two groups yields a biased estimator of the effect of the prerequisite on main course achievement. Yet it is reasonable to consider that students whose test scores are close to the cutoff score are similar. The idea that two groups whose scores are close to the cutoff score are similar is equivalent to the idea that they are similar to each other in terms of potential outcomes. It implies that the outcomes would be the same among the students who score close to the cutoff point in the assessment test, were it not for the assignment to the prerequisite course. It can be rephrased in the following assumption.

Assumption 1a. *i)* $E(Y_{i,1}|X_i = x)$ *is continuous in* x *at* c*, and ii)* $E(Y_{i,0}|X_i = x)$ *is continuous in* x *at* c

If it is true, the two groups whose test scores are close to the cutoff score are thought to be randomly assigned. Then the effect of the prerequisite can be identified by a comparison of outcomes between the two groups whose test scores are close to the cutoff. This is the main idea of regression discontinuity design.

Under the Assumption 1a, the effect of the prerequisite would be identified by the difference in the achievement in the main course between the students who score just below the cutoff and the students who score just above the cutoff.

$$E(\tau_i|X_i = c) = \lim_{x \uparrow c} E[Y_i|X_i = x] - \lim_{x \downarrow c} E[Y_i|X_i = x] \tag{2}$$

Without further assumptions of the common effect assumption, $\tau_i = \tau$ for all i, only at the cutoff score $x = c$ can treatment effects be identified. Compared to the randomization experiment, the disadvantage of a regression discontinuity design is that what can be known are only treatment effects near the cutoff score c.

3.1.1 Local Linear Regression Estimation

The estimation of equation (2) may be accomplished in various ways. The most often used estimators are global polynomial regressions (Black, Galdo, and Smith, 2007; Lee and Card, 2008; Lee and Lemieux, 2010)[16] and local linear regression (Hahn, Todd, and van der Klaauw, 2001; Porter, 2003; Imbens and Lemieux, 2008). These two estimation approaches are generally competitive, with differing strengths and weaknesses. Since the first approach is more sensitive to outcomes far from the cutoff than the second one, I use the second procedures to estimate the effect of its prerequisite on achievement in the main math course.

[16] Global polynomial regression estimations are also thought to be nonparametric as they are variants of series estimations (Lee and Lemieux, 2010).

Local linear regressions provide a nonparametric way of consistently estimating β_i in (2). According to Imbens and Lemieux (2008) who derive the special case of Hahn, Todd, and van der Klaauw (2001), the simple version of the local linear regression estimation can be presented. Define the conditional means on the left-hand side of x_0 in equation (3) and define the conditional means on the right-hand side of x_0 in equation (4).

$$\mu_l(x_0) = \lim_{x \uparrow x_0} E(Y_i | X_i = x) \tag{3}$$

$$\mu_r(x_0) = \lim_{x \downarrow x_0} E(Y_i | X_i = x) \tag{4}$$

Then, the estimand of interest is $E(\tau_i | X_i = c) = \mu_l(c) - \mu_r(c)$, denoted by τ.

I can fit linear regression functions to the observations within a given bandwidth h on either side of the discontinuity point $x = c$, applying rectangular or uniform kernel to Hahn, Todd, and van der Klaauw (2001)'s estimation.

$$\min_{\alpha_l, \beta_l} \sum_{i: c-h < X_i < c} (Y_i - \alpha_l - \beta_l(X_i - c))^2 \tag{5}$$

$$\min_{\alpha_r, \beta_r} \sum_{i: c \leq X_i < c+h} (Y_i - \alpha_r - \beta_r(X_i - c))^2 \tag{6}$$

The estimate of $\mu_l(c)$ is $\hat{\mu}_l(c) = \hat{\alpha}_l - \hat{\beta}_l(c - c) = \hat{\alpha}_l$, and the estimate of $\mu_r(c)$ is $\hat{\mu}_r(c) = \hat{\alpha}_r - \hat{\beta}_r(c - c) = \hat{\alpha}_r$. Then the estimated treatment effect is

$$\hat{\tau} = \hat{\alpha}_l - \hat{\alpha}_r \tag{7}$$

With the additional assumption of undersmoothing the bandwidth, $h \propto N^{-\delta}$ for $1/5 < \delta < 2/5$,

$$\sqrt{Nh}(\hat{\tau} - \tau) \longrightarrow \mathcal{N}\left(0, \frac{4(\sigma_l^2 + \sigma_r^2)}{f_X(c)}\right) \tag{8}$$

where $\sigma_l^2 = \lim_{x \uparrow c} Var(Y_i | X_i = x)$, and $\sigma_r^2 = \lim_{x \downarrow c} Var(Y_i | X_i = x)$, and f_X is a density function of X_i.

3.2 Missing Outcome Problem

The critical problem–the missing outcome problem–arises since a student's achievement in the main course can be observed only if that student completes the course. One of the reasons a student may not complete the course is withdrawal. The frequencies of withdrawal from the main course might not differ much between the treatment group and the control group if the sample is restricted to the students who enroll in the main course. Another reason for missing outcomes is that many students do not enroll in the main course so that their achievements Y_i in the main course cannot be observed. There could be a large difference in the proportion of enrollment in the main course between two groups, compared to small differences in the proportion of withdrawal. The students who are assigned to the prerequisite course are less likely to enroll in the main course even though most have successfully completed the prerequisite. The propensity to enroll in the main course can be said to systemically differ between the two groups. According to Lee (2009)'s general sample selection model, outcome observability can be modeled in Rubin's potential outcome setting, which can allow the treatment to cause the difference in the observability of the outcome between the treatment group and the control group. $S_{i,1}$ and $S_{i,0}$ are potential observability indicators for the treatment and control states, respectively. Denote the indicator of observability by S_i. Then the model in (1) can be presented in the following way.

$$Y_i = T_i Y_{1,i} + (1 - T_i) Y_{0,i} \tag{9}$$

$$S_i = T_i S_{1,i} + (1 - T_i) S_{0,i} \tag{10}$$

Y_i is observed if $S_i = 1$ or Y_i is missing if $S_i = 0$.

In addition to Assumption 1a, the continuity assumption for the observability is necessary for the identification in regression discontinuity design. It is given in the following.

Assumption 1b. *i) $E(S_{i,1}|X_i = x)$ is continuous in x at c, and ii) $E(S_{i,0}|X_i = x)$ is continuous in x at c*

This implies that the observability of the outcomes would be indifferent between the treatment group and the control group when the students in both groups score close to the cutoff point in the assessment test, were it not for the assignment to the prerequisite course.

The first estimand of interest is $E(\tau_i|X_i = c) = E(Y_{1,i} - Y_{0,i}|X_i = c)$, but it is impossible to identify it by the way of (2) when there exists the structural difference in the observability of the outcomes between two groups. Only with the indifference in observability, i.e, $S_i = S_{i,1} = S_{i,0}$, can the estimand be bounded via the method of Horowitz and Manski (2000), additionally assuming the boundedness of the outcome Y.

$$\tau_L \leq E(\tau_i|X_i = c) \leq \tau_U \tag{11}$$

$$\text{where } \tau_U = \lim_{x \uparrow c}[E(Y_i|X_i = x, S_i = 1)P(S_i = 1|X_i = x) + Y_{max}P(S_i = 0|X_i = x)] \tag{12}$$
$$- \lim_{x \downarrow c}[E(Y_i|X_i = x, S_i = 1)P(S_i = 1|X_i = x) + Y_{min}P(S_i = 0|X_i = x)]$$
$$\text{and } \tau_L = \lim_{x \uparrow c}[E(Y_i|X_i = x, S_i = 1)P(S_i = 1|X_i = x) + Y_{min}P(S_i = 0|X_i = x)] \tag{13}$$
$$- \lim_{x \downarrow c}[E(Y_i|X_i = x, S_i = 1)P(S_i = 1|X_i = x) + Y_{max}P(S_i = 0|X_i = x)]$$

For the same reason, the previous studies (McCrary and Royer, 2011; Martorell and McFarlin, 2011) which include the additive separable control function to handle the missing outcome problem in regression discontinuity design fail to identify the treatment effect when there exists a structural difference in the observability of the outcomes. Instead of using the potential outcome model (9) with the observability equation (10), they model the selection process as follows:

$$Y_i = \tau T_i + m(X_i) + U_i \tag{14}$$
$$S_i = \mathbf{1}(\rho T_i + n(X_i) + V_i \geq 0) \tag{15}$$
$$Y_i \text{ is observed if } S_i = 1 \text{ or } Y_i \text{ is missing if } S_i = 0.$$

Along the ways, as suggested by Heckman (1976, 1979), they assume the bivariate normality of (U_i, V_i) as for the exclusion restriction to the sample selection or the observability, and generate the control function, which is called the inverse Mill's ratio, and include it in the main model (14) to estimate the treatment effect τ. The control function is, however, discontinuous at the cutoff point when the observability of the outcomes varies structurally between the treatment group and the control group. Thus, their strategy using the exclusion restriction or the bivariate normality of (U_i, V_i) cannot identify the treatment effect if there exists systemic difference in the observability of the outcomes.

3.3 Bounding the Causal Effects

It is necessary to invoke the additional assumption to address the structural difference in the observability of outcomes. The students who are assigned to the prerequisite course are less likely to enroll in the main course than the students who are allowed to take the main course without the prerequisite, because most students try to avoid staying longer in school. As a result, assignment to the prerequisite course always reduces the observability of the outcome. This is summarized in the following assumption:

Assumption 2 (Monotonicity)**.** $S_{i,1} \leq S_{i,0}$ *with probability 1.*

It implies that treatment assignment can only affect observability in one direction. A student who is assigned to taking the prerequisite and completes the main course would enroll in the main course and complete it if he or she had no duty to take the prerequisite. Conversely, a student who is allowed to take the main course directly and completes it might not enroll in the course and thus fail to complete it if he or she had to take the prerequisite.

Invoking the monotonicity assumption (Assumption 2), the conditional expectations of the non-missing outcomes Y at the limit point at c both from below and from above can be shown in the following equations.

$$\begin{aligned}\lim_{x\uparrow c} E(Y|X=x, S=1) &= E(Y_1|X=c, S_1=1) \quad (16)\\ &= E(Y_1|X=c, S_1=1, S_0=1)\\ \lim_{x\downarrow c} E(Y|X=x, S=1) &= E(Y_0|X=c, S_0=1) \quad (17)\\ &= P(S_1=1|X=c, S_0=1)E(Y_0|X=c, S_1=1, S_0=1)\\ &+ P(S_1=0|X=c, S_0=1)E(Y_0|X=c, S_1=0, S_0=1)\end{aligned}$$

The limit from below in (16) exactly identifies the conditional mean of Y_1 on the one group $\{i : S_1 = 1, S_0 = 1\}$. Contrary to the limit from below, the limit from above in (17) cannot identify the outcomes of the one unique group. It is the mixture of the distributions of the two groups; $E(Y_0|X = c, S_1 = 1, S_0 = 1)$ and $E(Y_0|X = c, S_1 = 0, S_0 = 1)$.

First, the difference in the two limits will identify the treatment effect for the subgroup $\{i : S_1 = 1, S_0 = 1\}$, if $P(S_1 = 0|X = c, S_0 = 1) = P(S_1 = 0, S_0 = 1|X = c, S_0 = 1) = 0$. It would be the case, if the propensity to finish/complete the main course so that the outcome might be observable were the same irrespective of the assignment to the prerequisite course, $S_0 = S_1$ in probability 1. However, the probability of $\{i : S_1 = 0, S_0 = 1\}$ would be positive for those who score barely higher than the cutoff of the prerequisite. Among those students, some would not enroll in the main course if they were forced to take the prerequisite course first. Meanwhile, they would take the main course if they were allowed to take the main course without the prerequisite.

If it were possible to identify and discard that subgroup $\lim_{x\downarrow c}\{i : S_1 = 0, S_0 = 1, X = x\}$ from the control group $\lim_{x\downarrow c}\{i : S_0 = 1, X = x\}$, then the remaining would be $\lim_{x\downarrow c}\{i : S_1 = 1, S_0 = 1, X = x\}$, which would be comparable to the treatment group, $\lim_{x\uparrow c}\{i : S_1 = 1, X = x\} = \lim_{x\uparrow c}\{i : S_1 = 1, S_0 = 1, X = x\}$ at the cutoff point. However, it is impossible to identify and disentangle only the subgroup $\lim_{x\downarrow c}\{i : S_1 = 1, S_0 = 1, X = x\}$ from the control group. Moreover, it is important to note that only $E(\tau_i|S_1 = 1, S_0 = 1, X = c)$ can be identified at best, since the monotonicity assumption can identify only $\lim_{x\uparrow c}\{i : S_1 = 1, S_0 = 1, X = x\}$ from the treatment group.

For convenience, denote the probability of $P(S_1 = 0|X = c, S_0 = 1)$ by ϕ, and then $P(S_1 = 1|X = c, S_0 = 1) = 1 - \phi$. Note that ϕ can be identified by $\lim_{x\downarrow c} E(S|X = x)$ and $\lim_{x\uparrow c} E(S|X = x)$ from the data by the monotonicity assumption.

$$
\begin{aligned}
\phi = P(S_1 = 0|X = c, S_0 = 1) &= \frac{P(S_0 = 1, S_1 = 0|X = c)}{P(S_0 = 1|X = c)} \\
&= \frac{P(S_0 = 1|X = c) - P(S_0 = 1, S_1 = 1|X = c)}{P(S_0 = 1|X = c)} \\
&= \frac{P(S_0 = 1|X = c) - P(S_1 = 1|X = c)}{P(S_0 = 1|X = c)} \\
&= \frac{\lim_{x\downarrow c} E(S|X = x) - \lim_{x\uparrow c} E(S|X = x)}{\lim_{x\downarrow c} E(S|X = x)}
\end{aligned}
$$

ϕ is the proportion of the students whose outcomes in the main course are observable because of the assignment to the main course directly, but whose outcomes would not be observable if they were made to take the prerequisite course before the main course. In terms of Imbens and Angrist (1994), ϕ can be interpreted as the proportion of the marginal students who are induced to enroll in the main course and finish it to finally show their outcomes. The identification result and the form are also similar to their LATE's. Using the notation ϕ, the limit from above in (17) is expressed:

$$
\begin{aligned}
E(Y_0|X = c, S_0 = 1) = (1 - \phi)E(Y_0|X = c, S_1 = 1, S_0 = 1) \\
+ \phi E(Y_0|X = c, S_1 = 0, S_0 = 1)
\end{aligned} \tag{18}
$$

Recall that it is impossible to distinguish the two subgroups $\lim_{x\downarrow c}\{i : S_1 = 1, S_0 = 1, X = x\}$ and $\lim_{x\downarrow c}\{i : S_1 = 0, S_0 = 1, X = x\}$ from the control group without additional assumptions. Instead of invoking additional assumptions to separate those two subgroups from the control group, the extreme situation can be imagined. Consider the potential achievements Y_0 in the main course when the prerequisite course is not being taken. Without the help of the prerequisite course ($T = 0$), the potential achievements Y_0 of those who would always take the main course even with the restriction of taking the prerequisite course ($\lim_{x\downarrow c}\{i : S_0 = 1, S_1 = 1, X = x\}$) are always higher (or lower) than the maximum (or minimum) achievement in the main course of those who would not proceed to the main course if they scored barely less than the cutoff point and were assigned to the prerequisite restrictions ($\lim_{x\downarrow c}\{i : S_1 = 0, S_1 = 1, X = x\}$):

$$\inf\{Y_0|X = c, S_1 = 1, S_0 = 1\} \geq \sup\{Y_0|X = c, S_1 = 0, S_0 = 1\} \text{ with probability 1}$$
$$\text{or } \sup\{Y_0|X = c, S_1 = 1, S_0 = 1\} \leq \inf\{Y_0|X = c, S_1 = 0, S_0 = 1\} \text{ with probability 1}$$

Since the proportion ϕ of $\{i : X = c, S_1 = 0, S_1 = 1\}$ among the control group $\{i : X = c, S_1 = 1\}$ can be identified from the data, an upper bound for $E(Y_0|X = c, S_1 = 1, S_0 = 1)$ can be obtained, trimming the lower tail of the Y_0 distribution by the proportion ϕ. Similarly, a lower bound for $E(Y_0|X = c, S_1 = 1, S_0 = 1)$ can be obtained, trimming the higher tail of the Y_0 distribution by the proportion ϕ.

It is necessary to look at the distribution of the observed outcome Y of those students who score just above the cutoff c and are assigned to the main math, and find out the qth quantile, y_q; for a given q, $y_q = H^{-1}(q)$ with $H(y) = P(Y_0 \leq y|X = c, S_0 = 1)$[17]. Using the notation of the qth quantile, the upper bound and the lower bound for $E(Y_0|X = c, S_1 = 1, S_0 = 1)$ are to be obtained and they are proven to be sharp[18]. They are expressed in the following equations.

$$E(Y_0|X = c, S_0 = 1, S_1 = 1) \leq E(Y_0|X = c, S_0 = 1, Y_0 > y_{1-\phi})$$
$$E(Y_0|X = c, S_0 = 1, S_1 = 1) \geq E(Y_0|X = c, S_0 = 1, Y_0 \leq y_{\phi})$$

Consequently, both the lower bound τ_L and the upper bound τ_U for $E(\tau_i|X = c, S_1 = 1, S_0 = 1)$ are to be obtained, both of which are shown to be sharp.

$$\begin{aligned}
\tau_L &= E(Y_1|X = c, S_1 = 1, S_0 = 1) - E(Y_0|X = c, S_0 = 1, Y_0 > y_{1-\phi}) \\
&= \lim_{x \uparrow c} E(Y|X = x, S = 1) - \lim_{x \downarrow c} E(Y|X = x, S = 1, Y > y_{1-\phi}) \\
\tau_U &= E(Y_1|X = c, S_1 = 1, S_0 = 1) - E(Y_0|X = c, S_0 = 1, Y_0 \leq y_{\phi}) \\
&= \lim_{x \uparrow c} E(Y|X = x, S = 1) - \lim_{x \downarrow c} E(Y|X = x, S = 1, Y \leq y_{\phi})
\end{aligned}$$

Note that only $E(\tau_i|S_1 = 1, S_0 = 1, X = c) = E(Y_1 - Y_0|S_1 = 1, S_0 = 1, X = c)$ can be partially identified at best. The other parameters such as $E(\tau_i|X = c)$ and $E(\tau_i|S = 1, X = c)$ cannot be even partially identified with Assumption 1a through Assumption 2.

[17] This distribution is identified by $\lim_{x \downarrow c} P(Y \leq y|X = x, S = 1)$.

[18] Horowitz and Manski (1995) formally proves the expectation of the outcome after truncating the tails is the sharp upper or lower bound (Horowitz and Manski, 1995, Corollary 4.1), and Lee (2009) applies it to the context of missing outcome problems in the treatment effects.

3.4 Computation of Bounds by Local Linear Regression

Since the boundary problem appears in the application of the usual nonparametric kernel estimation to the regression discontinuity design, the estimation of both lower and upper bounds uses the local linear regression[19]

The estimation strategies are presented in the following. First, $\mu_{s,r}(c) = \lim_{x\downarrow c} E(S|X = x)$ and $\mu_{s,l}(c) = \lim_{x\uparrow c} E(S|X = x)$ are to be estimated by local linear regression.

$$(\hat{\alpha}_{s,r}, \hat{\beta}_{s,r}) = \underset{\alpha_{s,r},\beta_{s,r}}{\text{argmin}} \sum_{i:c\leq X_i<c+h} (S_i - \alpha_{s,r} - \beta_{s,r}(X_i - c))^2$$
$$(\hat{\alpha}_{s,l}, \hat{\beta}_{s,l}) = \underset{\alpha_{s,l},\beta_{s,l}}{\text{argmin}} \sum_{i:c-h\leq X_i<c} (S_i - \alpha_{s,l} - \beta_{s,l}(X_i - c))^2$$

The estimate of $\mu_{s,r}(c)$ is $\hat{\mu}_{s,r}(c) = \hat{\alpha}_{s,r}$, and the estimate of $\mu_{s,l}(c)$ is $\hat{\mu}_{s,l}(c) = \hat{\alpha}_{s,l}$. Then the estimator of ϕ can be obtained in the following way.

$$\hat{\phi} = \frac{\hat{\mu}_{s,r}(c) - \hat{\mu}_{s,l}(c)}{\hat{\mu}_{s,r}(c)} \tag{19}$$

Second, the $\hat{\phi}$th quantile and $(1 - \hat{\phi})$th quantile of the Y are to be estimated conditional on $S_0 = 1$ and around the cutoff $X = c$, i.e., restricting the sample to $\{i : c \leq X_i < c + h\}$, given the bandwidth h, which is used in the estimation of ϕ.

$$\hat{y}_{\hat{\phi},h} = \inf\{y : \hat{\phi} \leq \widehat{H}_h(y)\} \text{ with } \widehat{H}_h(y) = \frac{\sum_i \mathbf{1}(Y_i \leq y, c \leq X_i < c + h, S_i = 1)}{\sum_i \mathbf{1}(c \leq X_i < c + h, S_i = 1)} \tag{20}$$

Third, the estimands of the upper bound and the lower bound for $E(Y_0|X = c, S_1 = 1, S_0 = 1)$ are $\mu_{r,U}(c) = \lim_{x\downarrow c} E(Y|X = x, S = 1, Y > \hat{y}_{\hat{\phi},h})$ and $\mu_{r,L}(c) = \lim_{x\downarrow c} E(Y|X = x, S = 1, Y \leq \hat{y}_{1-\hat{\phi},h})$, respectively.

They are also estimated by local linear regression, using the same bandwidth h as in the estimation of ϕ.

$$\min_{\alpha_{r,U},\beta_{r,U}} \sum_{i:c\leq X_i<c+h,\ Y_i>\hat{y}_{\hat{\phi},h}} (Y_i - \alpha_{r,U} - \beta_{r,U}(X_i - c))^2$$
$$\min_{\alpha_{r,L},\beta_{r,L}} \sum_{i:c\leq X_i<c+h,\ Y_i\leq\hat{y}_{1-\hat{\phi},h}} (Y_i - \alpha_{r,L} - \beta_{r,L}(X_i - c))^2$$

The estimate of $\mu_{r,U}(c)$ is $\hat{\mu}_{r,U}(c) = \hat{\alpha}_{r,U}$, and the estimate of $\mu_{r,L}(c)$ is $\hat{\mu}_{r,L}(c) = \hat{\alpha}_{r,L}$.

[19] Lee (2009) shows the consistency and the asymptotic normality of the kernel estimators by the generalize moments methods.

Finally, the upper bound and the lower bound for the treatment effect τ are to be computed in the following way.

$$\hat{\tau}_U = \hat{\alpha}_l - \hat{\alpha}_{r,L} \tag{21}$$

$$\hat{\tau}_L = \hat{\alpha}_l - \hat{\alpha}_{r,U} \tag{22}$$

4 Data Description

4.1 One Community College (OCCSC)

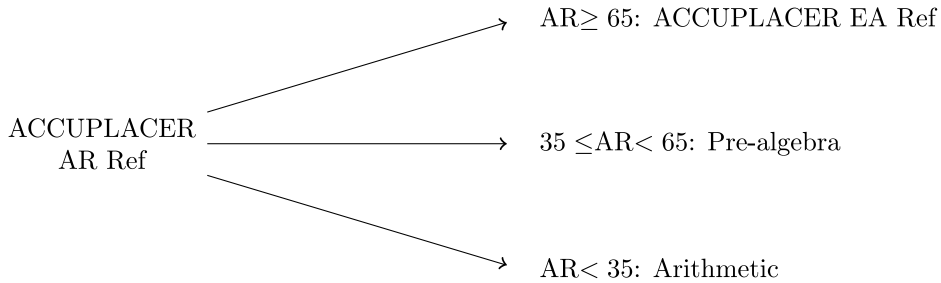
Unlike Florida and Texas (Calcagno and Long, 2008; Martorell and McFarlin, 2011, respectively), California has not maintained a single universal assignment policy across all the community colleges in the state, and hence a state-level analysis is impossible when using regression discontinuity design. Since each college in California has its own assignment policy, it is sensible to choose one community college when estimating the effect of developmental mathematics sequence by use of regression discontinuity design. The chosen college is located in an urban area of southern California; it is a large state institution with an annual freshman enrollment of around 3,000 students and an annual total enrollment of around 20,000 students. Thus it is called the one of community colleges in southern California (henceforth denoted as OCCSC).

All the students entering OCCSC are required to take the assessment test so that the administration can determine their level of mathematics skill. The level of mathematics course a student must take is determined by the cutoff points set up by OCCSC as well as her or his score on the assessment test. The assessment test used in OCCSC is the ACCUPLACER test developed by the College Board. The ACCUPLACER mathematics test is not a single-subject test. In particular, ACCUPLACER consists of three sub-categories: 1) an arithmetic test (ACCUPLACER AR), 2) an elementary algebra test (ACCUPLACER EA), and 3) a college-level mathematics test (ACCUPLACER CLM).

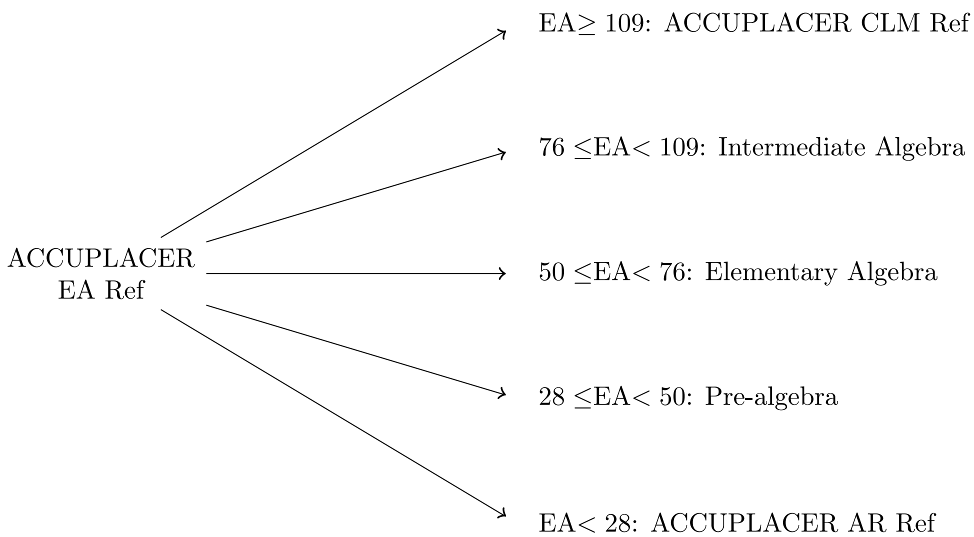
With the background questionnaire on an individual student, the computer administrative system chooses the beginning subject test for this student. Every student should begin the ACCUPLACER test in the one specific subject. Students might finish ACCUPLACER mathematics test in the same subject area as in the beginning, and be placed into some mathematics course. However, students sometimes proceed to another subject if their scores on the first subject test are too low or too high. As a result, they could take more than one subject test and finish the ACCUPLACER mathematics test in a subject area different from the beginning subject test.

Figure 1: Cutoff Policy of OCCSC between 2005/6 and 2007/8

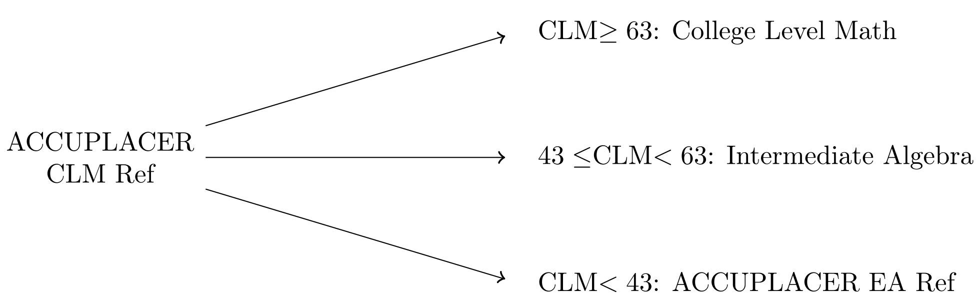
(a) Placement by ACCUPLACER AR Test



(b) Placement by ACCUPLACER EA Test



(c) Placement by ACCUPLACER CLM Test



Note: AR means the score on ACCUPLACER AR, EA means the score on ACCUPLACER EA, and CLM means the score on ACCUPLACER CLM. ACCUPLACER AR Ref means that a student is referred to taking ACCUPLACER AR test. ACCUPLACER EA Ref means that a student is referred to taking ACCUPLACER EA test. ACCUPLACER CLM Ref means that a student is referred to taking ACCUPLACER CLM test.

The assignment result depends on the score that a student receives in the last stage of the assessment test. Figure 1 shows the detailed cutoff policy that had been used between academic years 2005/6 and 2007/8 in OCCSC. From the fall semester of 2005 to spring 2008, the cutoff scores for the assignments had not been changed.

Note that multiple measure points which are calculated from the background questionnaire on students must be automatically added to all test scores in order to protect minorities. Multiple measure points are calculated based on the quantity and quality of high-school math which students previously took. The range of multiple measure points is from 0 to 4.

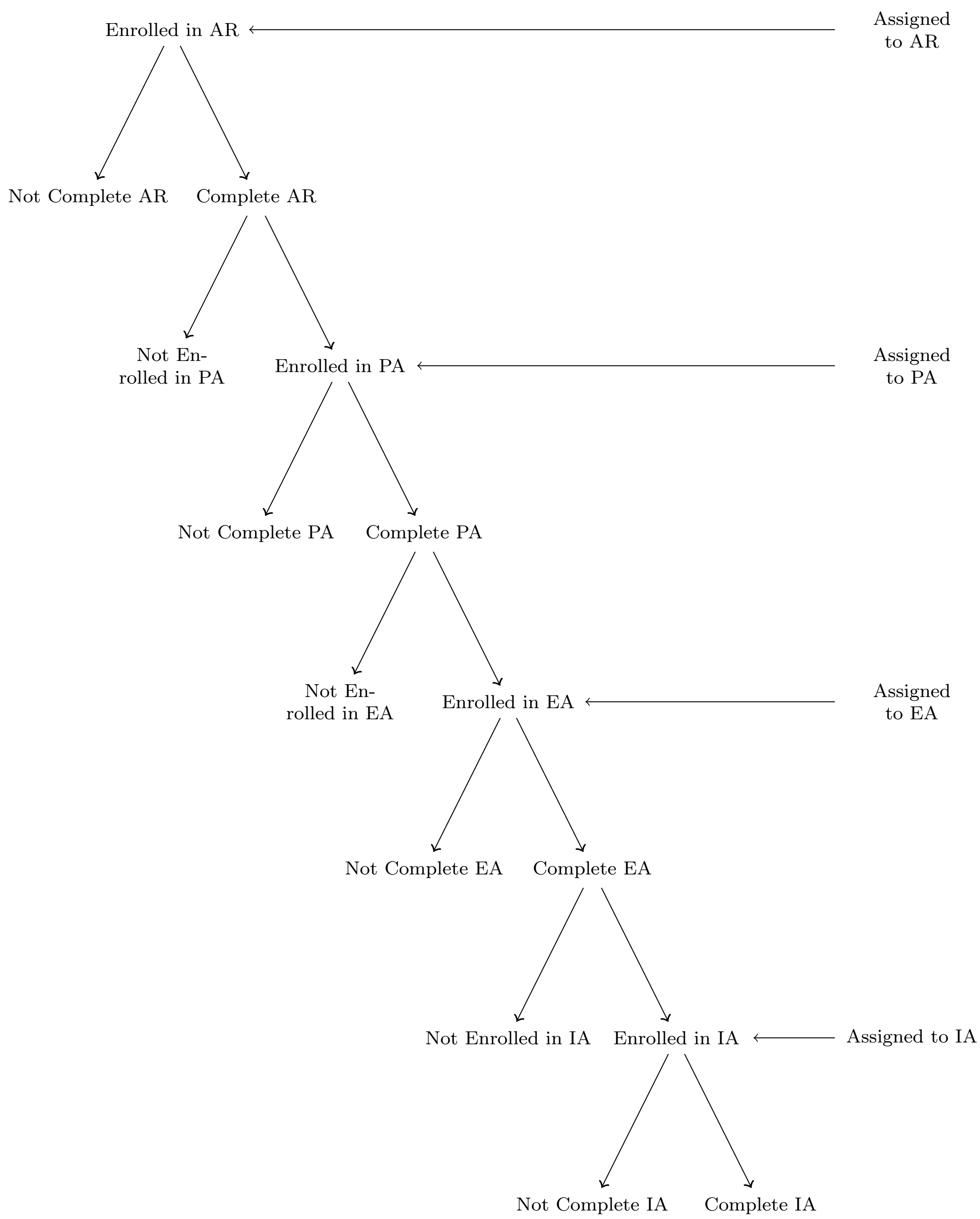
In OCCSC, a sequence of developmental math courses consists of 4 levels of mathematics as shown in Figure 2: 1) arithmetic, 2) pre-algebra, 3) elementary algebra, and 4) intermediate algebra. The description of each course in the sequence was already given in Section 2.2, and the aim of the developmental sequence is to help students be ready for college-level mathematics through the instruction of high-school-level courses. Not until a student completes the required intermediate algebra courses can he or she take any college-level math course as long as he or she is not assigned to college-level math. The rule of enrollment is that for taking the one specific course, a student should either complete its prerequisite course or be placed into that course.

4.2 Sample Criteria

I examine the students who took the assessment test between academic years 2005/6 and 2007/8 in OCCSC. In this period, the assessment policy was stable, and 10,874 students were assessed in the area of mathematics. 19% of students were assigned to arithmetic, 39% to pre-algebra, 20% to elementary algebra, 18% to intermediate algebra, and only 4% to college-level math. Since the study's main interest is in algebra courses, the sample was restricted to the students whose the last subject during the math assessment test was ACCUPLACER EA; 7,419 students were selected. If the student's last subject is ACCUPLACER EA during the assessment, then her/his scores on ACCUPLACER EA will assign a student to the one of algebra courses. When the outcomes of interest are elementary algebra's (intermediate algebra's), the corresponding treatment is assignment to pre-algebra (elementary algebra). Then the control group is the students who are assigned to elementary algebra (intermediate algebra), while the treatment group is the students who are assigned to pre-algebra (elementary algebra).

The first motive of this study is to examine whether a developmental math sequence in community colleges can make up for a lack of the mathematical skill that should have been imparted in domestic high schools. The students in the sample should have completed

Figure 2: A Sequence of Developmental Mathematics Courses in OCCSC



Note: AR means arithmetic course, PA means pre-algebra course, EA means elementary algebra course, and IA means intermediate algebra course.

the high-school mathematics sequence not in foreign countries but in the U.S. Moreover, the students in the sample are restricted to those of an age with the average college student. Their placement results can show the effectiveness of the high-school sequence offered recently, without depreciation in math knowledge. These restrictions impart meaning to the question of whether developmental mathematics in community colleges can help students catch up with their peers in four-year colleges. Students were excluded from the sample if 1) they were concurrent in high school and 2) they graduated from foreign high schools, or 3) they were older than 22 years old at the assessment.

Two additional but important criteria generate the final sample. The first criterion is to choose students who took the assessment test and enrolled in any math course. A student is said to participate in the developmental mathematics sequence if he or she enrolls in any math course. The students who did not enroll in any math course cannot have any meaningful outcome except the decision not to enroll in math. Although the assessment test was compulsory, the registration in OCCSC and its math program was up to an individual student's decision. There were many students who did not enroll in any math after the assessment test. Those who did not enroll in the math course, however, are not of interest because the aim of the developmental course is not to induce those to the developmental sequence but to develop mathematics skills for those who participate in the program.

The second criterion is to choose the students who did not retest. The primary reason is that for those who retested, it is difficult to construct the outcomes regarding the development sequence because they might stop taking the assigned course and then retest to be placed at a level higher than their first assignment. The rule is rigid on retesting. Without a strong excuse, a student cannot retake the assessment test within three years after the first assessment. Nonetheless, a few students retested despite the rule. If those students were more motivated than any others, excluding those students will create some bias in the estimation. The assignment results from the retests, however, were not different from the first results, and the individual characteristics of those who retested were not different from the ones of those who did not retest. Thus, excluding those who retested does not seem to create a bias.

4.3 Measures of Academic Achievement

Before presenting the descriptive statistics, it is necessary to define the appropriate outcomes in the study. The outcome of interest is achievement in the one mathematics course. Regarding the measurement of student's achievement, however, one important problem arises. There are no standardized end-of-course tests, and hence no standardized measure of achievement.

Instead, grade point averages (GPA) on a course would be used as a measure of academic achievement in a course. The first reason to use its average as the measure of achievement is that a course might consist of two semesters; e.g., elementary algebra consists of Math 113 and 114 in OCCSC. The other reason is that in many cases a student repeats taking a course. The average points adjust the waste of time involved in repeating the course. Calculating the GPA on a course includes the letter grade of failure as well as the letter grade of withdrawal.

Another measure is the time (or semesters) to complete the main course. Completing the main course means that a student gets at least D on all the courses of the main course. If a course consists of two semester courses, the completion of a course means the completion of both semester courses. The time to complete the main course can measure the efficiency of producing meaningful achievement from the main course.

Note that the GPA on the main course cannot be seen if a student withdraws from it or if he or she does not enroll. It can be observable only if he or she finishes at least one course and gets a letter grade including F. Similar to GPA, the time to complete the main course can be observable only if a student completes it or obtains at least D in all the courses of the main course. The enrollment in the main course is related to the observability of the outcomes of the main course. Unless a student enrolls in the course, he or she can neither finish nor complete it.

4.4 Descriptive Statistics

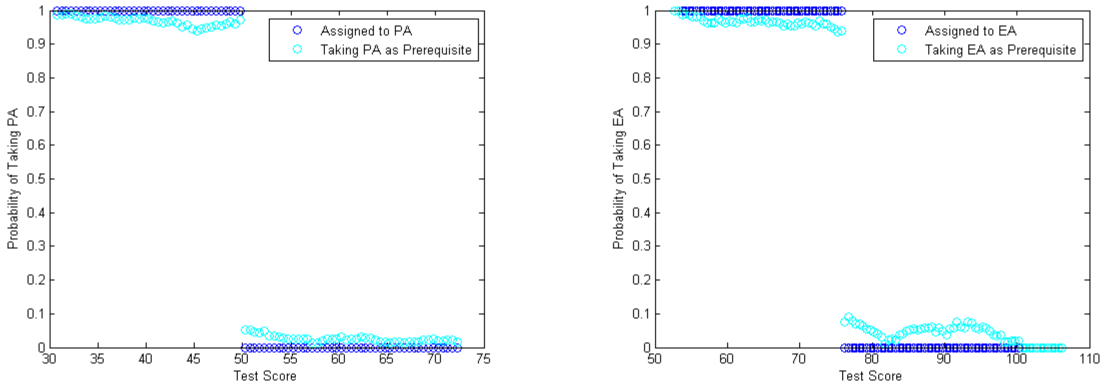
Table 1 reports descriptive statistics of the selected sample. After selecting the sample by the criteria summarized above, the number of the sample is reduced to 2,483. The most significant reason for the decrease in the sample size is that many students did not enroll in any math course. The first column of the table reports all the students whose last subject during the assessment test was ACCUPLACER EA. These students were assigned to one of three algebra courses: 1) pre-algebra, 2) elementary algebra, and 3) intermediate algebra. The second column corresponds to the students who were placed in pre-algebra and the third column corresponds to those in elementary algebra. The final column describes those whose assignment results are intermediate algebra.

Three important features of the data are worth mentioning. First, the lower the levels to which students are assigned, the worse their outcomes. Second, students assigned to the lower level are with from backgrounds; they are more likely to be African American or Hispanic, and they have lower multiple measure points. But it cannot be said that the assignment itself cause the results. Rather, the students with lower baseline characteristics

Table 1: Descriptive Statistics

	All	Assigned to PA	Assigned to EA	Assigned to IA
Age at the Assessment	19.0	19.1	18.9	19.0
	(1.2)	(1.2)	(1.1)	(1.2)
Female	0.55	0.58	0.55	0.49
Black/Hispanic	0.71	0.79	0.71	0.52
Non U.S. Citizen	0.28	0.24	0.30	0.33
English is NOT Primary Language	0.42	0.43	0.41	0.44
Test Score	56.6	38.5	62.5	90.3
	(20.9)	(5.9)	(7.2)	(10.7)
Multple Measure Points	2.28	2.11	2.36	2.57
	(0.86)	(0.82)	(0.85)	(0.87)
Assigned to PA	0.47			
Assigned to EA	0.34			
Assigned to IA	0.19			
Enroll in the Assignment	0.96	0.96	0.96	0.95
Enrolled in PA	0.46	0.96		
Finish PA	0.37	0.76		
Mean Grade† on PA	1.45	1.43		
	(1.27)	(1.27)		
Complete PA	0.26	0.54		
Semesters‡ to Complete PA	1.25	1.26		
	(0.52)	(0.52)		
Enrolled in EA	0.54	0.42	0.96	
Finish EA	0.43	0.33	0.76	
Mean Grade† on EA	1.59	1.38	1.69	
	(1.20)	(1.13)	(1.22)	
Complete EA	0.32	0.25	0.57	
Semesters‡ to Complete EA	1.42	1.57	1.34	
	(0.76)	(0.90)	(0.66)	
Enrolled in IA	0.43	0.19	0.47	0.95
Finish IA	0.34	0.14	0.39	0.74
Mean Grade† on IA	1.63	1.44	1.60	1.75
	(1.14)	(1.05)	(1.14)	(1.17)
Complete IA	0.27	0.11	0.32	0.59
Semesters‡ to Complete IA	1.36	1.38	1.41	1.30
	(0.68)	(0.71)	(0.74)	(0.59)
Number of Observations	2483	1157	851	475

Note: Table reports means and standard deviations which are shown in parentheses for the entering students who were assessed between 2005/6 and 2007/2008 in OCCSC, and their last subject during the assessment test was elementary algebra. See text for details of sample selection. †: The mean grade on the course can be obtained if a student finishes it or gets a letter grade on it. ‡: The semester to complete the course can be obtained if a student completes it.



(a) Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)

(b) Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

Figure 3: The Proportion of the Assignment to the Prerequisite Courses and the Enrollment in the Prerequisite Assignment

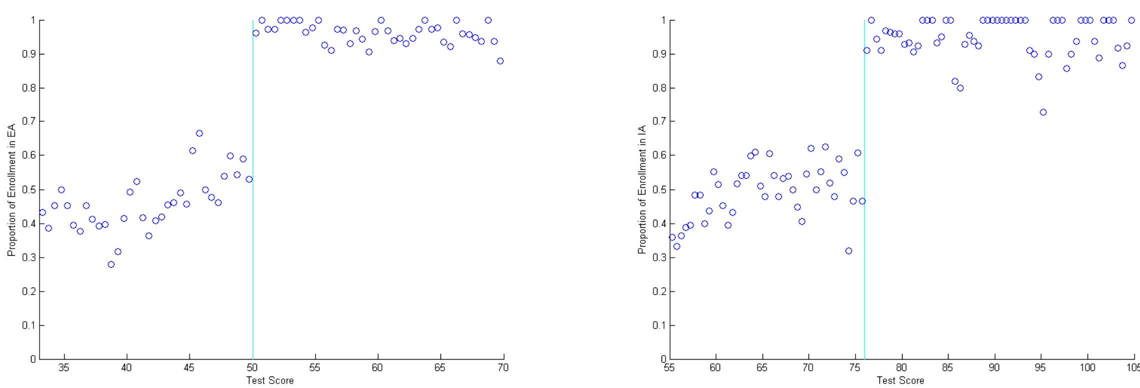
produce worse outputs, and assignment status is correlated with these factors. Finally, the observability in the outcomes varies among the three groups. Their mean grades on the main course can be observable only if students finish it and get letter grades, while their time to complete the main course can be observable only if students complete it. Thus, the indicators of finishing the course and of completing the course are observability indicator variables for its mean grades and the time to complete it, respectively. The lower the level of the assigned course is, the less its propensity to be observable is. Relating to the observability of the outcomes, enrollment in the main course is important. The rate of enrollment in the main course also shows the same patterns as the finishing rate and the completion rate.

Description statistics shows that almost every student (95%) followed the assignment result. Among the assignment statuses there are no differences in the likelihood of complying with the assignment results. In addition, Figure 3 shows the proportion of the students who were assigned to the prerequisite course and the students who actually took that course as a prerequisite. The assignment results seem to perfectly align with the placement rules, while compliance with the assignment results does not seem to be perfect. However, very few students did not follow the course assignments and it is thought to be all right to regard compliance with the assignments as almost perfect. Contrary to the other studies of community colleges using regression discontinuity design (Calcagno and Long, 2008; Martorell and McFarlin, 2011), I do not have to use fuzzy regression discontinuity and instrumental variables defined by the assignment in order to control noncompliance problem. Only the results based on sharp regression discontinuity will be shown.

5 Results

5.1 Differences in Enrollment

Enrollment in a course is an important indicator of the observability of achievement, though not all those who enrolled in a course finished/completed it. Figure 4 plots the likelihood of enrollment in the main course for two cases: 1) where pre-algebra is a prerequisite to elementary algebra and 2) where elementary algebra is a prerequisite to intermediate algebra. Both cases have the same result: the rate of the enrollment in the main course is discontinuous at the cutoff point between the prerequisite course assignment and the main course assignment. Half of those assigned to the prerequisite course do not enroll in the main course. Because the relationship between test scores and enrollment rates looks very flat except for the cutoff point, test scores themselves do not seem to affect the likelihood of enrollment in the main course. The difference in the enrollment can be due to only the difference in the course assignments.



(a) Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)

(b) Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

Figure 4: The Proportion of Enrollment in the Main Course

Table 2 reports the estimated prerequisite assignment effects on the enrollment in the main course. $\hat{\mu}_{s,l}$ (or $\hat{\mu}_{s,r}$) is the estimate of the proportion of enrollment in the main course for those assigned to the prerequisite (or those directly assigned to the main). All the estimates are obtained by local linear regression. The difference $\hat{\mu}_{s,l} - \hat{\mu}_{s,r}$ is the estimate of the causal effect of the prerequisite course assignment. Each column presents a different bandwidth used in local linear regression, and reports its corresponding result.

An important issue is the choice of the smoothing parameter, the bandwidth h. There are many automatic bandwidth selectors for nonparametric regression, but two methods

Table 2: Estimated Difference in the Enrollment in the Main Course between the Group Assigned to the Prerequisite and the Group Assigned Directly to the Main.

A. The main course is elementary algebra (EA) The prerequisite is pre-algebra (PA)	(1) CV	(2) ROT	(3) Medium
Bandwidth	17.5	2.9	10
$\hat{\mu}_{s,r}$	0.976	0.959	0.991
$\hat{\mu}_{s,l}$	0.530	0.523	0.562
$\hat{\mu}_{s,l} - \hat{\mu}_{s,r}$	-0.446***	-0.436***	-0.429***
Standard Error	0.038	0.100	0.053

B. The main course is intermediate algebra (IA) The prerequisite is elementary algebra (EA)	(1) CV	(2) ROT	(3) Medium
Bandwidth	11.4	2.7	7
$\hat{\mu}_{s,r}$	0.951	0.921	0.935
$\hat{\mu}_{s,l}$	0.517	0.522	0.514
$\hat{\mu}_{s,l} - \hat{\mu}_{s,r}$	-0.433***	-0.399***	-0.421***
Standard Error	0.064	0.133	0.084

* indicates the 10% significance, ** indicates the 5% significance , *** indicates the 1% significance level. Note: $\mu_{s,r} = \lim_{x \downarrow c} E(S|X = x)$ is the fraction of the students who enroll in the main course among those who score barely above the cutoff so that they do not have to take the prerequisite course (control group). $\mu_{s,l} = \lim_{x \uparrow c} E(S|X = x)$ is the fraction of the students who enroll in the main course among those who score barely below the cutoff so that they must take the prerequisite course (treatment group). Those estimators $\hat{\mu}_{s,l}$ and $\hat{\mu}_{s,r}$ are obtained by local linear regression in (5) and (6), respectively. $\hat{\mu}_{s,l} - \hat{\mu}_{s,r}$ is difference in the fraction of the students who enroll in the main course, and it measures how many students in the margin do not enroll in the main course because of the assignment to the prerequisite course. Each column corresponds to the method to obtain the bandwidth h. In column (1), h is obtained by the modified cross validation (CV) method suggested by Imbens and Lemieux (2008) and Ludwig and Miller (2005), discarding the 95% of observations in tails. In column (2), h is obtained by the rule of thumbs (ROT) derived by Fan and Gijbels (1996), assuming the rectangular kernel. In column (3), h is arbitrarily set. Standard errors of $\hat{\mu}_{s,l} - \hat{\mu}_{s,r}$ are estimated by (8)

are used here. The first is to use Ludwig and Miller (2005) and Imbens and Lemieux (2008)'s modified cross-validation procedure. Modified cross-validation procedure discards observations close to both tails, when calculating the cross-validation criterion. It chooses the optimal bandwidth h_{opt}, which minimizes the modified cross-validation criterion. I discard 95% of observations when choosing the optimal bandwidth h_{opt}. The second is a simple automatic procedure that Fan and Gijbels (1996, Section 4.2) provide. This procedure fits a fourth-order global polynomial separately on the left and the right of the cutoff point. For either side, the rule-of-thumb (ROT) bandwidth is $c\left(\frac{\hat{\sigma}^2(\max\{X_i\}-\min\{X_i\})}{\sum_i m''(X_i)}\right)^{\frac{1}{5}}$, where $m''(X_i)$ is the estimated second derivative of the global polynomial evaluated at X_i, $\hat{\sigma}^2$ is the mean squared error for the regression, $\max\{X_i\} - \min\{X_i\}$ is the range of X_i, and a constant $c = 2.702$ is specific to the rectangular kernel used here. Between two ROT bandwidths, I choose the smaller one.

The results are not only robust to the choice of bandwidth, but also to the kinds of courses. If a student was assigned to pre-algebra, he or she was 43 – 45% less likely to enroll in elementary algebra than a student who could enroll in it directly. Similarly, a student who was assigned to elementary algebra was 40 – 43% less likely to take intermediate algebra than a student assigned to intermediate algebra. In addition, the flatness of the conditional expectation of the enrollment in the main course implies that once a student was assigned to the prerequisite course he or she was 40 – 50% less likely to take the main course irrespective of test scores and the kind of prerequisite.

It can be inferred that students do not enroll in the next-level course just due to the requirement of the prerequisite in itself. Whenever developmental math courses are differentiated and sequentially organized, the same problems always occur. The main course's outcomes cannot be observed for some of those assigned to the prerequisite course all the time. The way of addressing the missing outcome problems is salient in the evaluation of developmental mathematics offered at community colleges.

5.2 Main Results: GPA on the Main Course

I now turn to the results for main outcomes, the GPA on the main course. Figure 5 shows the proportion of finishing the main courses and the conditional expectation of mean grade on the main course. There is evidence that the rate of finishing the main course is also discontinuous at the cutoff point between the prerequisite course and its subsequent course, as seen in Figures 5a and 5b. Figure 5c shows that those who barely failed the cutoff score and hence were assigned to the prerequisite course pre-algebra surpassed the counterpart assigned to elementary algebra directly. But Figure 5d finds no discontinuity at the cutoff point between elementary algebra and intermediate algebra Note that Figures 5c and 5d can plot only the observable outcomes, and thus the shown discontinuity could overestimate or underestimate the true effects on the GPA on the main course.

Table 3 reports the estimates of the effect of the assignment to the prerequisite course on the GPA on the subsequent main course. Two estimation procedures are used. Panel I has the result of the local linear regression estimation by conditioning on the observable GPA on the main course. Panel II reports the lower (or upper) bound for the treatment effects by discarding some portion of highest (or lowest) outcomes of the control group. The procedure in panel I corresponds to Figures 5c and 5d and it serves as the benchmark to the bounding procedure in panel II, though the first yields biased estimates due to the sample selection problem.

There are two issues to be discussed before presenting the results. First, the choice of bandwidth is not yet clear, because the same bandwidth should be used for the estimation of ϕ and the computation of the bounds, τ_L and τ_U. The optimal bandwidth can be attained from either the estimation of the effect on observability or the estimation of the treatment effect on the outcome by conditioning on the observable outcomes. The curvatures are different between two outcomes, so the corresponding bandwidths are different. I choose the derived bandwidth from the estimation of the effect on the outcome. It is reasonable to think that the curvature of the true outcomes is more similar to the one of observable outcomes than the curvature of the observability indicators despite the possible bias.

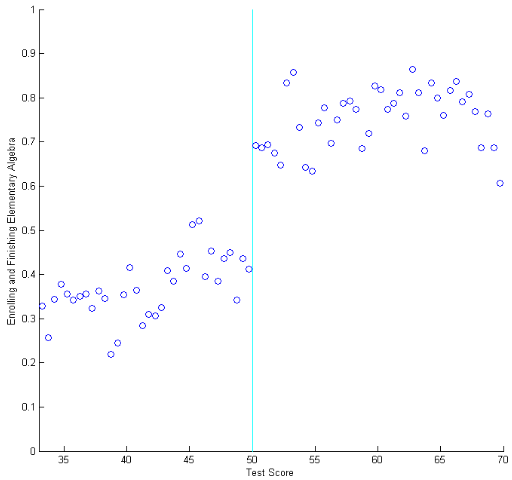
The second issue is the inference on the treatment effects as well as the bounds. Imbens and Manski (2004) suggest the way to compute a 95% confidence interval for the parameter of interest, the effect of the prerequisite course on the GPA on the next course. The interval of $[\hat{\tau}_L - \bar{C}_n \frac{\widehat{\sigma_{\tau_L}}}{\sqrt{n}}, \hat{\tau}_U + \bar{C}_n \frac{\widehat{\sigma_{\tau_U}}}{\sqrt{n}}]$ contains the parameter $E(Y_1 - Y_0|S_1 = 1, S_0 = 1, X = c)$ with a probability of at least 0.95, where n is the sample size, $\widehat{\sigma_{\tau_L}}$ and $\widehat{\sigma_{\tau_U}}$ are the standard errors of the lower bound and the upper bound, respectively, and $\bar{C}_n$ satisfies

$$\Phi\left(\bar{C}_n + \frac{\sqrt{n}(\hat{\tau}_U - \hat{\tau}_L)}{\max(\widehat{\sigma_{\tau_L}}, \widehat{\sigma_{\tau_U}})}\right) - \Phi\left(-\bar{C}_n\right) = 0.95$$

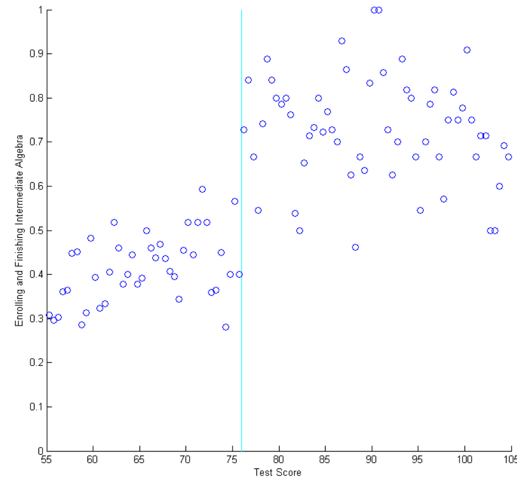
But the variances $\sigma^2_{\tau_L}$ and $\sigma^2_{\tau_U}$ are not discussed, though the identification and estimation of the bounds τ_L and τ_U are shown in Section 3. Instead of deriving the analytic asymptotic variances[20], bootstrapping is used to estimate the variance of the bounds (Horowitz, 2001; Horowitz and Manski, 2000). When bootstrapping the standard error of the bounds,

[20]This approach is unattractive because the expressions for the asymptotic variance are very lengthy and thus tedious to implement. Even the simplest case with no covariates is very complicated (See Lee, 2009, Proposition 3 and its proof).

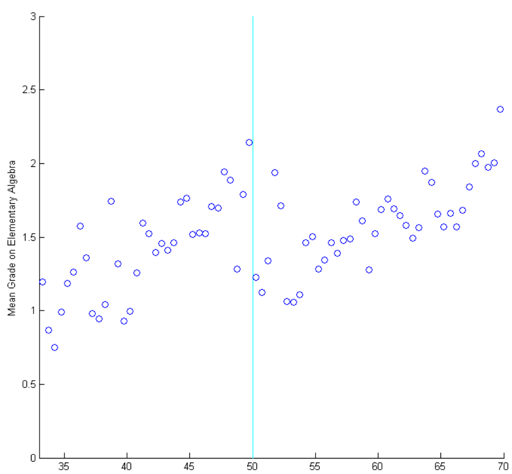
Figure 5: Finishing the Main Course and Mean Grade on the Main Course



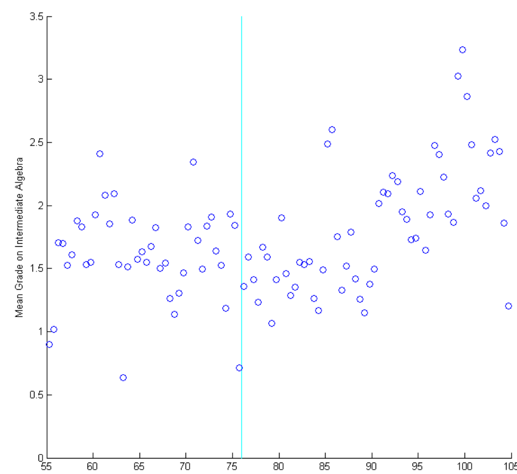
(a) Finishing the Main Course
Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(b) Finishing the Main Course
Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)



(c) Mean Grade on the Main Course
Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(d) Mean Grade on the Main Course
Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

sampling is done at the level of the test score X_i, given the bandwidth.

The left side of panel I in Table 3 contains the benchmark results for the impact of pre-algebra on the GPA on elementary algebra. Even when one restricts the sample to the students who finished elementary algebra, pre-algebra seems to help to improve the skill of students in elementary algebra. Note that this estimate can exaggerate the effect if those assigned to pre-algebra did not enroll in elementary algebra because they were believed to be inferior in math. Otherwise, it can be biased in the downward direction. There is no telling whether the estimated effects in panel I are overestimated or underestimated from the given data and assumptions.

Instead of point identification of the effects, I employ bounding procedures to compute the effect of pre-algebra in panel II. The upper bound estimates means that the assignment to pre-algebra would increase the GPA on elementary algebra by 0.95 – 1.3 points in the best case. The lower bound estimates implies that there are no significant effects on elementary algebra in the worst case. The estimated proportion $\hat{\phi}$ implies that 33 – 44% of those who were assigned to elementary algebra and finished it would not finish elementary algebra if they were required to take pre-algebra. Some would drop off the developmental program because they thought that they had already mastered pre-algebra and that taking pre-algebra would be a waste of time. In contrast, some would exit the developmental math sequence because they were not confident of passing pre-algebra. The lower bound corresponds to the case where all the students in $\lim_{x \downarrow c}\{i : S_0 = 0, S_1 = 1, X = x\}$ are of the first type, while the upper bound corresponds to the case where all the students in $\lim_{x \downarrow c}\{i : S_0 = 0, S_1 = 1, X = x\}$ are of the second type. It is the extreme case that $\lim_{x \downarrow c}\{i : S_0 = 0, S_1 = 1, X = x\}$ consists of only one type. In particular, the lower bounds can be realized only if all the students who would not finish elementary algebra were assigned to pre-algebra, and its corresponding estimate looks like almost zero. As a result, the true effect can be significantly positive, though it can be lower than not only the upper bound but also the estimates from conditioning on the observable outcomes.

Contrary to the effect of pre-algebra, the estimated effects of elementary algebra on the mean grade on intermediate algebra are easy to interpret. First, restricting the sample to the students who finish intermediate algebra, it appears that elementary algebra does not raise intermediate algebra skill. Second, the lower-bound estimates are significantly negative and the upper-bound estimates are significantly positive, irrespective of the choice of bandwidth. In addition, the median value of both bounds is close to zero. It implies that the effect of elementary algebra is much more likely to be insignificant.

Table 3: Effects of Prerequisite Course on the Average Grade Points of the Main Course

I. Local Linear Regression Estimation, Conditioning on the Observable Outcomes.						
	A. The main course is EA The prerequisite is PA			B. The main course is IA The prerequisite is EA		
	(1) CV	(2) ROT	(3) Medium	(1) CV	(2) ROT	(3) Medium
Bandwidth	11.6	4.7	8	12	4.6	8
$\hat{\tau}$	0.687***	0.676**	0.664***	0.166	0.082	0.201
	(0.210)	(0.325)	(0.251)	(0.245)	(0.406)	(0.312)
N	1022	1022	1022	684	684	684

II. Local Linear Regression Estimation of the Bounds						
	A. The main course is EA The prerequisite is PA			B. The main course is IA The prerequisite is EA		
	(1) CV	(2) ROT	(3) Medium	(1) CV	(2) ROT	(3) Medium
$\hat{\phi}$	0.334***	0.460***	0.344***	0.381***	0.435***	0.405***
	(0.076)	(0.115)	(0.094)	(0.085)	(0.157)	(0.104)
$\hat{\tau}_L$	-0.007	-0.030	-0.055	-0.685**	-0.922	-0.725*
	(0.258)	(0.478)	(0.321)	(0.328)	(0.735)	(0.416)
$\hat{\tau}_U$	0.966***	1.530***	0.947**	0.763***	0.905**	1.048***
	(0.271)	(0.520)	(0.371)	(0.252)	(0.427)	(0.346)
Lower 95%	-0.021	-0.072	-0.076	-0.707	-1.001	-0.759
Upper 95%	0.980	1.576	0.972	0.780	0.950	1.077
N	2008	2008	2008	1326	1326	1326

* indicates the 10% significance, ** indicates the 5% significance , *** indicates the 1% significance level. Note: Standard errors are reported in parentheses. In the panel I, τ is the effect of the prerequisite on the main course, conditioning on the students who finish the main course. It is estimated by local linear regression in (5), (6) and (7). Its standard error is calculated by (8). In column (1), h is obtained by the modified cross validation (CV) method suggested by Imbens and Lemieux (2008) and Ludwig and Miller (2005), discarding the 95% of observations in tails. In column (2), h is obtained by the rule of thumbs (ROT) derived by Fan and Gijbels (1996), assuming the rectangular kernel. In column (3), h is arbitrarily set. In the panel II, the estimated bounds and their relevant results are reported. The same bandwidths as in the panel A are used for the estimation of the bounds. ϕ is the proportion of the students who finish the main course because of the assignment to the main course directly, but would not finish the main course if they were ordered to take the prerequisite before the main course. It is estimated by the local linear estimates of $\hat{\mu}_{s,l}$ and $\hat{\mu}_{s,r}$ defined in (19). τ_L and τ_U are the lower and upper bound for the effect of prerequisite course on the average grades on the main course, restricting to the students who always finish the main course irrespective of the assignment result. Both are estimated by (21) and (22) and their preceding procedures. Standard errors of $\hat{\phi}$, $\hat{\tau_L}$, and $\hat{\tau_U}$ are calculated via 500 bootstrap replications, where sampling is done at the test score level.

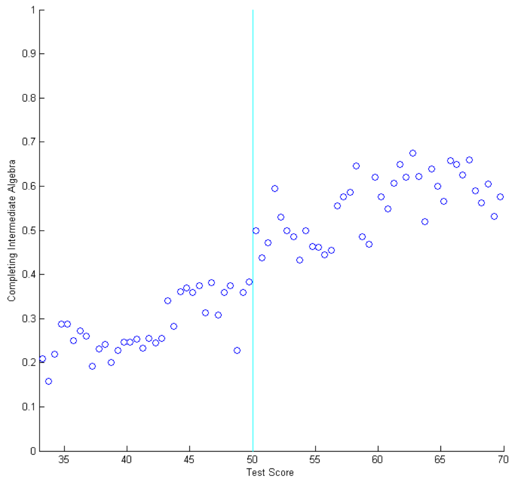
5.3 Main Results: Time to Complete the Main Course

The second main outcome is the time to complete the main course. The time is measured in terms of semesters. This outcome might reflect learning efficiency. Figure 6 shows the conditional expectation of time (semesters) to complete the main course and the proportion of observable outcomes, i.e., the proportion of completion of the main courses. Compared to the rate of finishing the main course, the rate of completion of the main course looks less discontinuous at the cutoff point, as seen in Figures 6a and 6b. Even if the proportion of getting letter grades on the main course is discontinuously higher for those directly assigned to the main course, the likelihood that those students finally obtain positive grades on the main course is not so high. It implies that students who are assigned to prerequisite courses are more likely to get positive grades on the main courses, at least when the sample is restricted to students who finish the main course or get the letter grade. Both Figures 6c and 6d give no visible evidence of the discontinuity at the cutoff point between the prerequisite and the main course. Note that Figures 6c and 6d can only plot the observable outcomes, and thus the shown graphs may not reflect the true patterns.

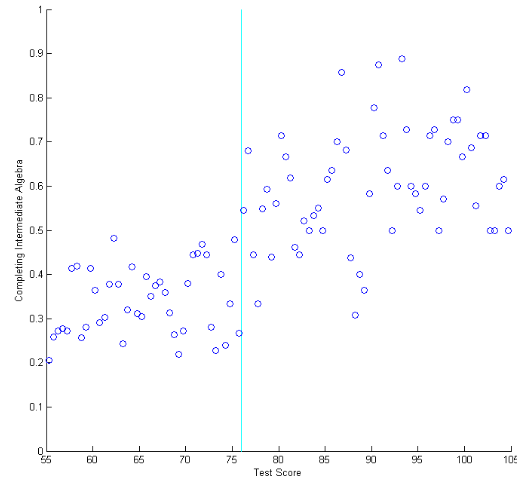
Table 4 reports the effects of the prerequisite course on the time to complete the main course. Panel I has the result of the local linear regression estimation, conditioning on the observable time to complete the main course. This panel corresponds to Figures 6c and 6d. The estimates in panel I may give biased estimates due to the sample selection problem. Panel II reports the lower (or upper) bound for the treatment effects discarding the highest (or lowest) outcomes of the control group.

First, I look at the effect of the pre-algebra course on the time to complete elementary algebra. Contrary to the effects on the GPA, this estimation's results are sensitive to the choice of bandwidths when restricting to observable outcomes. In the case of estimation of pre-algebra's effect on elementary algebra, the estimate conditioning on the observable outcomes is from -0.65 to -0.51 when using the optimal bandwidth selected by the rule-of-thumb and the cross-validation procedure. Meanwhile, the estimate using large bandwidths is not significant. Their bounds are less sensitive to choice of bandwidth. Upper bounds are always insignificant. Although the variation in the lower bounds' magnitude is a little larger than in the case of estimating the effect on the mean grades, the lower bounds are also significantly negative. Applying the same reasoning as in interpreting the bounds of the effects on the GPA, the assignment to pre-algebra seems to reduce the time to complete elementary algebra. In particular, the computed bounds and 95% intervals belong to the negative region, when using the ROT chosen bandwidth. This result supports the conclusion that pre-algebra would reduce the time to complete elementary algebra, and hence accelerate

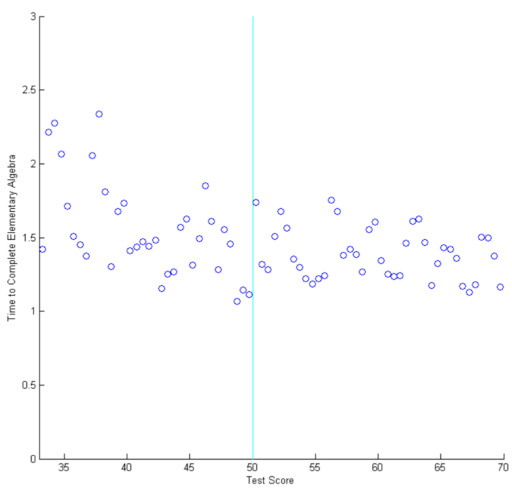
Figure 6: Completion of the Main Course and Time to Complete the Main Course



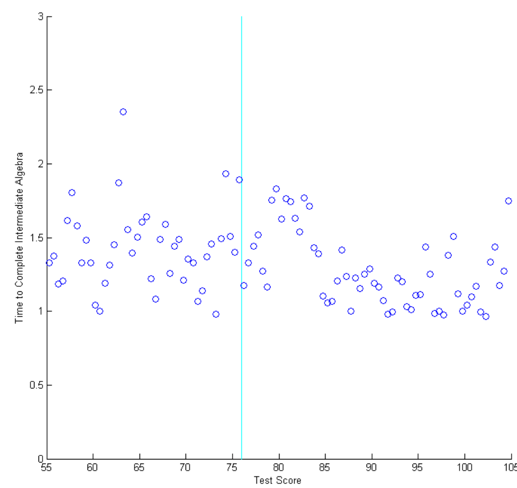
(a) Completion of the Main Course
Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(b) Completion of the Main Course
Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)



(c) Time to Complete the Main Course
Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(d) Time to Complete the Main Course
Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

Table 4: Effects of Prerequisite Course on the Time to Complete the Main Course

I. Local Linear Regression Estimation, Conditioning on the Observable Outcomes.						
	A. The main course is EA The prerequisite is PA			B. The main course is IA The prerequisite is EA		
	(1) CV	(2) ROT	(3) Large	(1) CV	(2) ROT	(3) Large
Bandwidth	5.3	2.6	8	7.9	2.5	12
$\hat{\tau}$	-0.512**	-0.652**	-0.141	0.260	0.329	-0.110
	(0.219)	(0.314)	(0.179)	(0.228)	(0.415)	(0.180)
N	775	775	775	548	548	548

II. Local Linear Regression Estimation of the Bounds						
	A. The main course is EA The prerequisite is PA			B. The main course is IA The prerequisite is EA		
	(1) CV	(2) ROT	(3) Large	(1) CV	(2) ROT	(3) Large
$\hat{\phi}$	0.349**	0.234	0.233*	0.269*	0.388	0.283**
	(0.155)	(0.344)	(0.135)	(0.160)	(0.287)	(0.137)
$\hat{\tau}_L$	-0.988***	-1.099***	-0.703**	-0.414	0.236	-0.798*
	(0.253)	(0.412)	(0.279)	(0.382)	(0.993)	(0.328)
$\hat{\tau}_U$	0.165	-0.386	0.105	0.518**	0.729	0.344*
	(0.265)	(0.351)	(0.257)	(0.229)	(0.465)	(0.203)
Lower 95%	-1.009	-1.147	-0.722	-0.445	0.093	-0.820
Upper 95%	0.187	-0.345	0.122	0.537	0.796	0.357
N	2008	2008	2008	1326	1326	1326

* indicates the 10% significance, ** indicates the 5% significance , *** indicates the 1% significance level.
Note: Standard errors are reported in parentheses. In the panel I, τ is the effect of the prerequisite on the main course, conditioning on the students who complete the main course. It is estimated by local linear regression in (5), (6) and (7). Its standard error is calculated by (8). In column (1), h is obtained by the modified cross validation (CV) method suggested by Imbens and Lemieux (2008) and Ludwig and Miller (2005), discarding the 95% of observations in tails. In column (2), h is obtained by the rule of thumbs (ROT) derived by Fan and Gijbels (1996), assuming the rectangular kernel. In column (3), h is arbitrarily set. In the panel II, the estimated bounds and their relevant results are reported. The same bandwidths as in the panel A are used for the estimation of the bounds. ϕ is the proportion of the students who complete the main course because of the assignment to the main course directly, but would not complete the main course if they were ordered to take the prerequisite before the main course. It is estimated by the local linear estimates of $\hat{\mu}_{s,l}$ and $\hat{\mu}_{s,r}$ defined in (19). τ_L and τ_U are the lower and upper bound for the effect of prerequisite course on the time to complete the main course, restricting to the students who always complete the main course irrespective of the assignment result. Both are estimated by (21) and (22) and their preceding procedures. Standard errors of $\hat{\phi}$, $\hat{\tau}_L$, and $\hat{\tau}_U$ are calculated via 500 bootstrap replications, where sampling is done at the test score level.

learning it.

Second, the effects of elementary algebra on time to complete intermediate algebra are examined. The estimates conditioning on the observable outcomes are always insignificant regardless of bandwidth. However, the estimated bounds are sensitive to the choice of bandwidth. Using the bandwidth (2.5) by ROT, elementary algebra seems to rather increase the time to complete intermediate algebra. Even though the computed bounds are insignificantly different from zero, 95% intervals belong to the positive area. Using the bandwidth (7.9) from the cross-validation method, the upper bound is significantly positive and the lower bound is insignificant[21] But the large-sized bandwidth gives the opposite results. The estimates by use of the above bandwidths imply that elementary algebra assignment would reduce the time to complete the main course. Thus, the effect of the assignment to elementary algebra on the time to complete intermediate algebra cannot be concluded now.

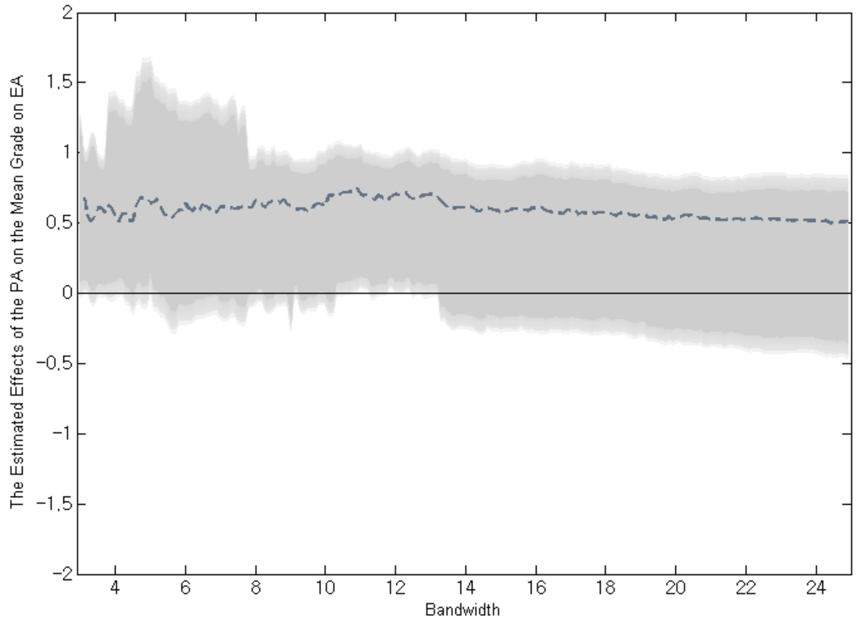
5.4 Sensitivity to the Choice of Bandwidths

Using the all available bandwidths, I check the estimates' robustness to the choice of bandwidths. Figure 7 shows the estimated effects on the GPA on the main course along the line of the bandwidth. The estimated bounds are stable for both i) the pre-algebra assignment's effect on the GPA on elementary algebra and ii) the elementary algebra assignment's effect on the GPA on intermediate algebra. Pre-algebra would increase the GPA on elementary algebra, while elementary would not significantly increase the GPA on intermediate algebra. There is no difficulty in interpreting the results for the prerequisite effects on the GPA on the main course.

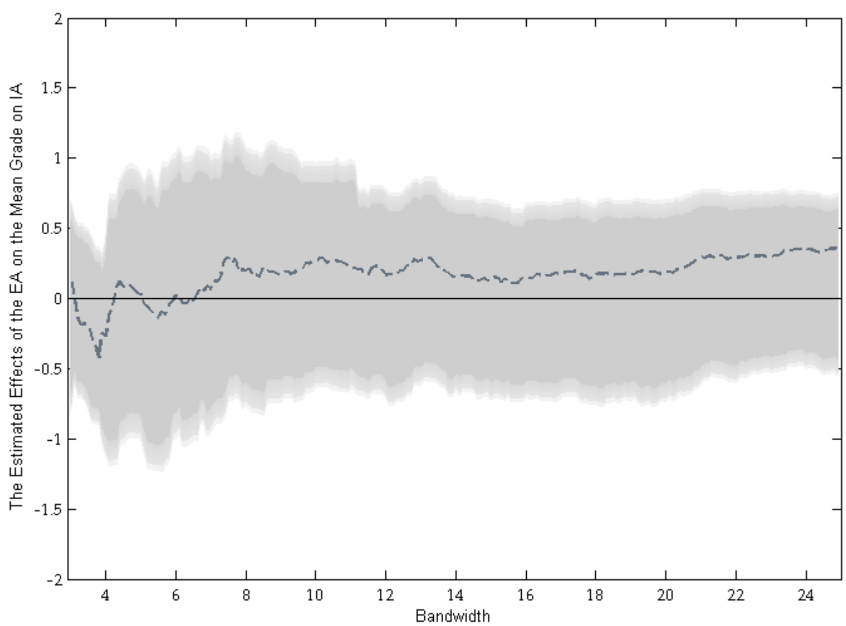
Figure 8 shows the estimated effects on the time or semesters it took for students to complete the main course along the line of the bandwidths. It is notable that the estimated effects are very strong when using the tiny bandwidths: i) the effects of the assignment to pre-algebra on elementary algebra are strongly negative, and ii) the effects of the assignment to elementary algebra on intermediate algebra are strongly positive. But when using the wider bandwidths, the significance looks smaller and smaller. Another noticeable point is that the sign of the effect of elementary algebra on the time to complete intermediate algebra flips when the size of bandwidth exceeds around 9. Contrary to the estimation of the effect on the GPA, the prerequisite effects on the time to complete the main course are more sensitive to the bandwidth choice since the estimation in regression discontinuity design is a limit at the cutoff point.

[21] The median of two bounds seems to be zero, and hence it implies no significant effects of the elementary algebra.

Figure 7: Bandwidths and the Estimated Prerequisite Effect on the Mean Grade

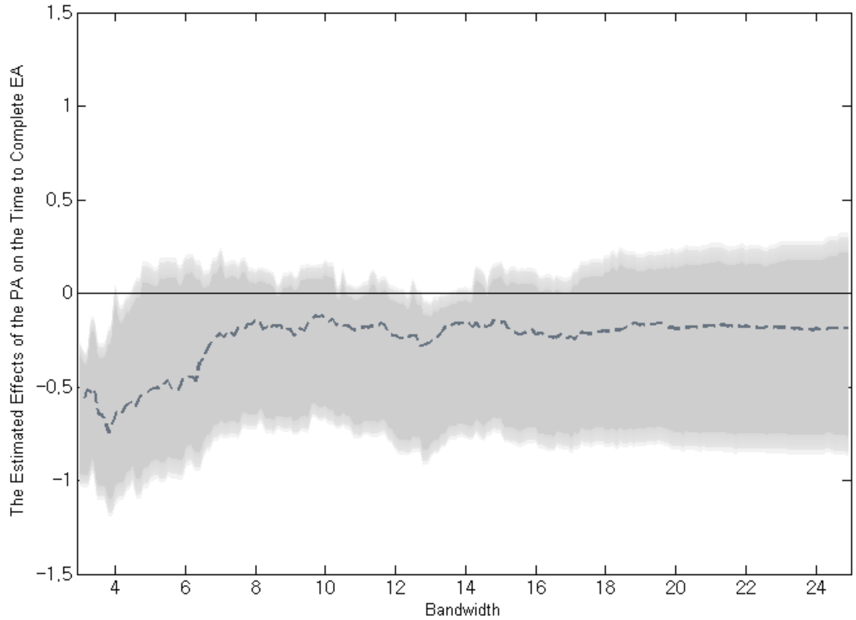


(a) Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)

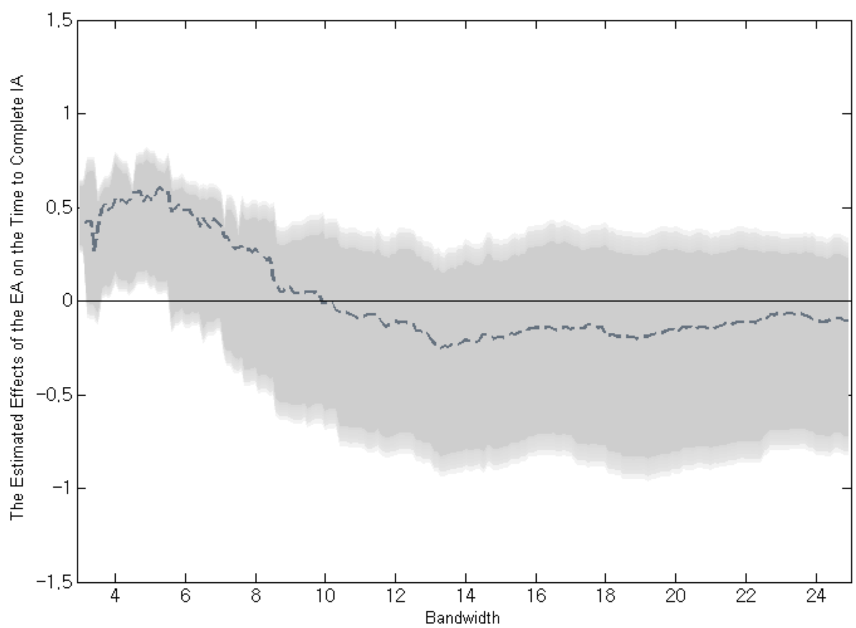


(b) Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

Figure 8: Bandwidths and the Estimated Prerequisite Effect on the Time to Complete



(a) Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(b) Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

The reason is that the curvatures of the conditional expectations of the time to complete main course change drastically along the test scores, as seen in Figures 6c and 6d. Despite the sensitivity to the bandwidth choice, the preferred results are the computed bounds which smaller bandwidths give. Thus, it follows that pre-algebra would increase the efficiency of learning elementary algebra, by reducing the time to complete elementary algebra by a half semester. However, the assignment to elementary algebra would slow down learning intermediate algebra.

6 Validity of Regression Discontinuity Design

So far I have estimated the effect of the prerequisite on achievement in the subsequent course. The estimation results are reliable as long as Assumptions 1a and 1b hold. Two assumptions imply that students who are assigned to the prerequisite and students who are not required to take the prerequisite are similar in all the other characteristics that determine achievement outcomes when one restricts the sample to students who were close to the cutoff point for the prerequisite course. Then the assumptions required for the validity of regression discontinuity design have two testable hypotheses: (1) the conditional expectation of preset covariates and (2) the density of the test scores should be continuous at the cutoff (Lee, 2008; McCrary, 2008). In this section, I will test these important hypotheses.

6.1 Discontinuities in Covariates

The continuity Assumptions 1a and 1b implies that all predetermined characteristics at the time of the assessment should be similar for the groups of students barely failing and barely passing the cutoff. Although it is impossible to test those assumptions directly, Lee (2008) derives the necessary condition by invoking additional assumptions. While it cannot be certain that the unobservable characteristics of students are continuous at the cutoff point, the validity of this assumption can be tested by ensuring that the conditional expectations of the observable characteristics do not vary discontinuously in the neighborhood of the cutoff score. Table 5 reports the estimated discontinuities in the covariates available from the data set, using the same local linear regression procedures as treatment effects being estimated. All the bandwidths correspond to the ones in Table 3 and Table 4.

First, look at the case in which pre-algebra is assigned as the prerequisite course and elementary algebra is the main course. The estimated discontinuities for most of the covariates are tiny, as seen in the table. It is particularly notable that multiple measure points are nearly identical. Multiple measure points are calculated based on the quantity and quality

Table 5: Estimated Discontinuity in Covariates

	Discontinuity at the Cutoff Point between Pre-algebra and Elementary Algebra						Discontinuity at the Cutoff Point between Elementary Algebra and Intermediate Algebra					
	Bandwidth						Bandwidth					
Variable	(1) 11.6	(2) 4.7	(3) 8	(4) 5.3	(5) 2.6	(6) 8	(1) 12	(2) 4.6	(3) 8	(4) 7.9	(5) 2.5	(6) 12
Age at the Assessment	0.08	-0.05	-0.07	-0.04	-0.15	-0.07	0.03	0.29	0.03	0.05	0.24	0.03
	(0.15)	(0.26)	(0.19)	(0.24)	(0.35)	(0.19)	(0.19)	(0.30)	(0.23)	(0.23)	(0.43)	(0.19)
Female	0.12*	0.13	0.13	0.14	0.02	0.13	-0.04	-0.08	-0.05	-0.05	-0.15	-0.04
	(0.06)	(0.11)	(0.08)	(0.10)	(0.14)	(0.08)	(0.08)	(0.13)	(0.10)	(0.10)	(0.18)	(0.08)
Black/Hispanic	-0.03	0.03	-0.04	0.05	-0.02	-0.04	-0.08	-0.09	-0.04	-0.05	0.05	-0.08
	(0.06)	(0.09)	(0.07)	(0.08)	(0.12)	(0.07)	(0.08)	(0.13)	(0.10)	(0.10)	(0.17)	(0.08)
Non U.S. Citizen	-0.04	-0.06	-0.09	-0.09	-0.04	-0.09	0.09	0.08	0.07	0.08	0.22	0.09
	(0.06)	(0.09)	(0.07)	(0.09)	(0.12)	(0.07)	(0.07)	(0.12)	(0.09)	(0.09)	(0.16)	(0.07)
English is NOT Primary	0.03	-0.05	-0.03	-0.04	-0.12	-0.03	0.00	0.13	0.01	0.01	0.23	0.00
Language	(0.06)	(0.10)	(0.08)	(0.10)	(0.14)	(0.08)	(0.08)	(0.13)	(0.10)	(0.10)	(0.18)	(0.08)
Multple Measure Points	-0.01	0.10	-0.01	0.10	0.11	-0.01	-0.22*	-0.43**	-0.30*	-0.32**	-0.71**	-0.22*
	(0.11)	(0.18)	(0.14)	(0.17)	(0.26)	(0.14)	(0.14)	(0.21)	(0.16)	(0.16)	(0.31)	(0.14)
Number of Observation	2008						1326					

* indicates the 10% significance, ** indicates the 5% significance , *** indicates the 1% significance level.
Note: Standard errors are reported in parentheses. All the estimated discontinuities are obtained by local linear regression. The corresponding bandwidths come from Table 3 and Table 4.

of high-school math which students previously took. Unlike the other covariates, multiple measure points are academic performance measures related to community-college math. If they are discontinuous at the cutoff point between pre-algebra and elementary algebra, the estimated effect of the pre-algebra cannot be reliable because of the confoundedness. Moreover, particular attention should be paid to multiple measure points since they are directly added to the test score (ACCUPLACER) to form the adjusted test score, which is actually used for course placement. No difference in multiple measure points around the cutoff ensures that the assignment to pre-algebra is locally randomized by the placement policy and the test score (ACCUPLACER).

For those who are close to the cutoff point between elementary algebra and intermediate algebra, all the covariates are continuous at the cutoff point except the most important covariate: multiple measure point. Those assigned to elementary algebra have (0.29 – 0.36 points) smaller multiple measure points, compared to those who were assigned to intermediate algebra without the requirement of elementary algebra. It means that those assigned to elementary algebra are significantly less likely to complete either algebra 2 or beyond-algebra math. From the standpoint of measuring outcomes in regression discontinuity design, this difference in multiple measure points poses a potential problem. It implies that the estimated effect of elementary algebra on intermediate algebra outcomes might not be unbiased. The estimated effect captures the effect of difference in achievement in high-school math between two groups. It is interesting how much this discontinuity biases the estimates of the effect of elementary algebra on the outcomes in the subsequent math course. It is connected with the cause of the discontinuity. I discuss the cause of the discontinuity and the direction of bias due to this continuity after checking the discontinuity in the density of test scores.

6.2 Jumps in the Distribution of the Test Scores

As emphasized by Lee (2008) and McCrary (2008), manipulation of test scores can be a critical threat to the validity of regression discontinuity design. Assignment to the prerequisite course around the pass-fail threshold would be randomized as long as test scores cannot be perfectly manipulated by students, teachers, etc. In the present case the manipulation seems quite unlikely, because the test used in OCCSC is ACCUPLACER. ACCUPLACER is a computerized test system, so there is no chance to manipulate test scores. There would be an observable discontinuity in the density of baseline test scores at the cutoff if there were the manipulation of the test scores. I test for a discontinuity in the density function of

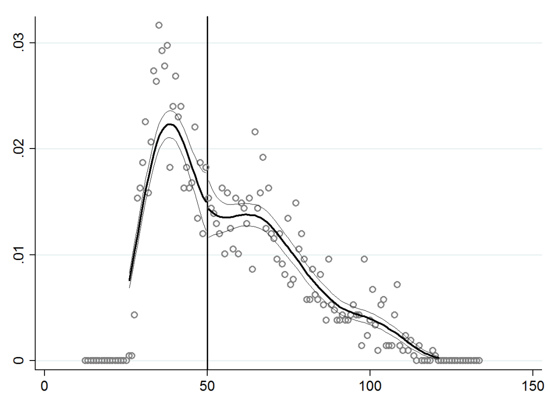
Table 6: McCrary Manipulation Test Log Discontinuity Estimates

A. Students Enrolled in Math, who were selected to the sample in Table 1		
	Discontinuity at the Cutoff Point between Pre-algebra and Elementary Algebra	Discontinuity at the Cutoff Point between Elementary Algebra and Intermediate Algebra
Estimates	0.019	-0.313*
Standard Errors	0.144	0.194
Bin Size	0.84	0.84
Bandwidth	12.83	10.71
Number of Observation	2483	

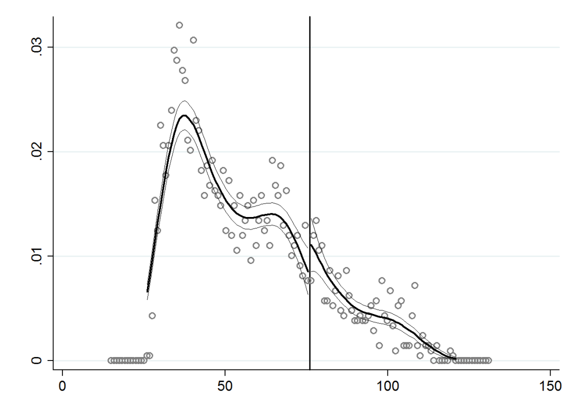
B. All Students who were assesed and whose last test was ACCUPLACER EA		
	Discontinuity at the Cutoff Point between Pre-algebra and Elementary Algebra	Discontinuity at the Cutoff Point between Elementary Algebra and Intermediate Algebra
Estimates	0.027	0.014
Standard Errors	0.098	0.115
Bin Size	0.54	0.54
Bandwidth	10.92	10.61
Number of Observation	7419	

* indicates the 10% significance, ** indicates the 5% significance , *** indicates the 1% significance level.
Note: Standard errors are reported in parentheses, and they are derived from McCrary (2008).

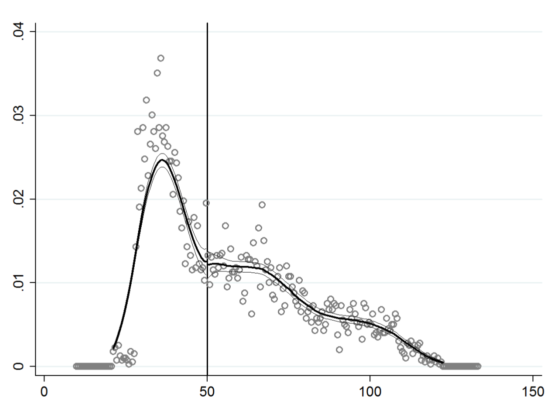
Figure 9: Density of the Test Scores



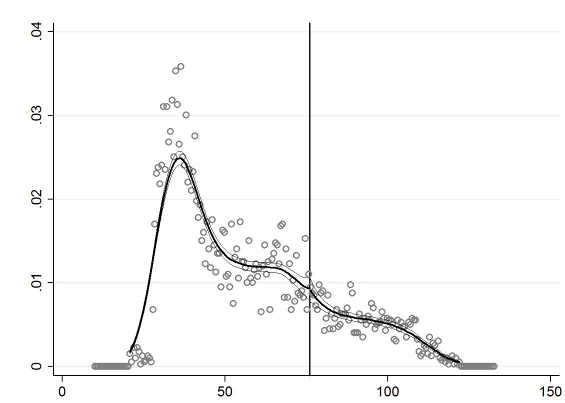
(a) Students Enrolled in Math
Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(b) Students Enrolled in Math
Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)



(c) All Assessed Students
Prerequisite: Pre-algebra (PA)
Main: Elementary Algebra (EA)



(d) All Assessed Students
Prerequisite: Elementary Algebra (EA)
Main: Intermediate Algebra (IA)

the ACCUPLACER EA test[22] using a test proposed by McCrary (2008), which is called the McCrary test.

Table 6 confirms that no statistically significant discontinuities are evident in the log of the test score densities for the ACCUPLACER EA test for the case where the prerequisite is pre-algebra and the main is elementary algebra. Figure 9 shows the corresponding pictures of the log of the density of ACCUPLACER EA. Note that, however, there is a noticeable difference in the log of the density, when the sample is restricted to the students who stand between elementary algebra and intermediate algebra; the log of the density of the test scores of students who score just below the cutoff score, 76, is remarkably smaller than that of their counterparts. This is seen in Figure 9b.

But it cannot be said that this discontinuity results from the manipulation of test scores. Rather, it is the result of restricting the sample to the students who participate in the developmental math program offered at OCCSC. For the unrestricted sample, all the students who were assessed at OCCSC, the McCrary test shows no discontinuity at the cutoff between those who were assigned to elementary algebra and those who were not required to take elementary algebra, in Figure 9d and in the right column of Table 6's panel B.

6.3 Discussion of Discontinuities in Multiple Measure Points and Density of Test Scores

The discontinuities in 1) multiple measure points and 2) the density of test scores (ACCUPLACER) are found at the cutoff between elementary algebra assignment and intermediate algebra. Now I examine why this discontinuity occurs when I restrict the sample to the students who barely pass or fail the cutoff point between elementary algebra and intermediate algebra. The conjecture is that among the students whose test scores are close to the cutoff point between elementary algebra and intermediate algebra, those who completed higher-level math in high school but are assigned to elementary algebra are more likely to be out of the sample than any others, when sample is restricted to those who participate in the developmental math sequence offered at OCCSC.

Consider three students. Two students completed all the algebra courses and beyond in high school and have the same high multiple measure points. One student did not complete them and hence has no multiple measure points. Between the two students who completed higher-level math, 1) one student barely passes the cutoff so that he or she is assigned to intermediate algebra, while 2) another barely fails so that he or she must take elementary

[22] Actually, the test score for the placement rule is the sum of test scores from ACCUPLACER test and multiple measure points.

algebra before intermediate algebra. And 3) the student who did not complete higher-level math is assigned to elementary algebra. If the second student is significantly likely to refuse to participate in the developmental mathematics sequence while the first and the third participate in the sequence, the discontinuity in multiple measure points can be seen. As a result, the density of test scores can be seen to be discontinuous at the cutoff point, too.

These discontinuities appear only when assigning students to elementary algebra or intermediate algebra. The first reason is that intermediate algebra or algebra 2 is the optional course in high school, unlike elementary algebra or algebra 1 in California. Those who completed intermediate algebra in high school are inclined to be proud of their completion of the optional and higher-level course. If they barely fail so that they are assigned to elementary algebra, they are too discouraged to participate in the developmental math sequence. The second reason is that those who are close to the cutoff point between pre-algebra and elementary algebra may be more homogeneous than those who are close to the cutoff point between elementary algebra and intermediate algebra. Since multiple measure points can be acquired only if higher algebra and beyond is taken, those who stand between pre-algebra and elementary algebra obtain almost zero multiple measure points. And those might not be disappointed though they were assigned to pre-algebra.

If the above statement can justify the discontinuity in multiple measure points, these discontinuities can explain the insignificance of the estimated effects of elementary algebra on the subsequent intermediate algebra's outcomes. The estimated effects of elementary algebra on the GPA on intermediate algebra (or the time to complete it) are downward (or upward) biased. Among the students assigned to elementary algebra, the students who completed algebra 2 are more likely to refuse the developmental math sequence or community-college education than any others. Those who are out of the sample can be considered to have higher potential to make up for their lack of knowledge of intermediate algebra, or to develop their skills. Incidentally, those students back out, and hence the placement policy based on the test score fails the local randomization in regression discontinuity design. As a result, the insignificant effects are unreliable when investigating elementary algebra's effect on the subsequent math course, intermediate algebra.

7 Conclusion

This paper focuses on an important issue in evaluating the effects of developmental mathematics: the missing outcome problem. It is a serious issue even when the assignment to the prerequisite course is believed to be locally randomized close to the cutoff point for the assignment to the prerequisite and the assignment to no prerequisite. Existing sample-selection correction methods are not feasible due to discontinuity in the observability of the outcomes. In order to estimate the effects of the prerequisite course on achievement in the subsequent course, this paper tries to bound treatment effects in a regression discontinuity designin the presence of missing-outcome problems. An appealing feature of the method is that the assumptions for identification and continuity of potential outcomes are typically already assumed in standard models of regression discontinuity design. Additional assumption of monotonicity of observability reflects the properties of developmental education offered at community colleges. In the case of regression discontinuity design, the continuity assumption is shown to be satisfied in the check of the validity of regression discontinuity designas illustrated in the previous section.

The analysis using the proposed bounds points to two substantive conclusions about the developmental mathematics offered at community colleges. First, the evidence shows that assignment to developmental courses should increase the student's GPA on the subsequent math courses. Second, prerequisite courses should reduce the time to complete the subsequent courses, and increase learning efficiency. Thus, the magnitudes found in this analysis of developmental mathematics support the opinion that developmental mathematics can help students who are unprepared for college-level math make up for their lack of skill in high-school mathematics.

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